Energy harvesting with nonlinear piezoelectric (μ)oscillators

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Noise in Physical Systems

NANOPOWER

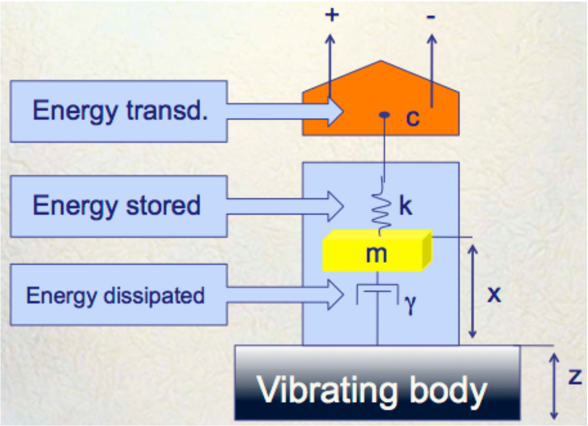
ZEROPOWER

Vibrations energy harvesting

The model:

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - c(x,V) + \zeta_z \\ \dot{V} = F(\dot{x},V) \end{cases}$$

Details depend on the physics…



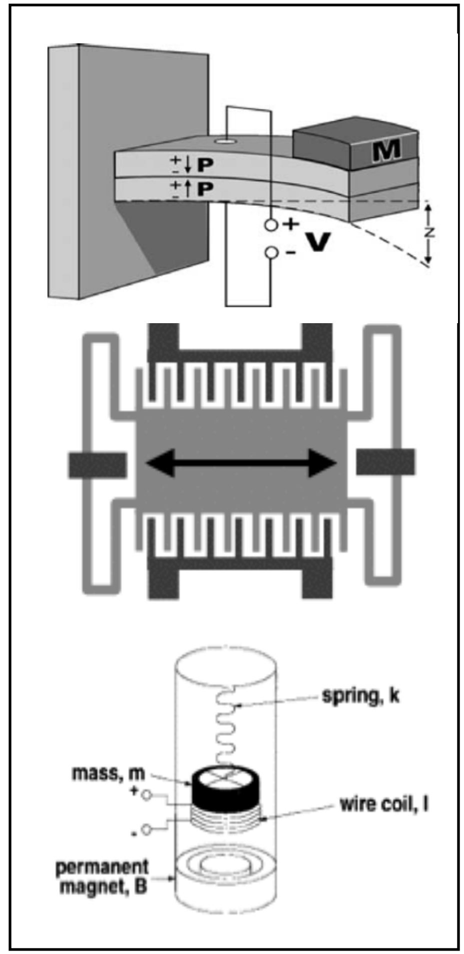
Equations that link the vibration-induced displacement with the Voltage

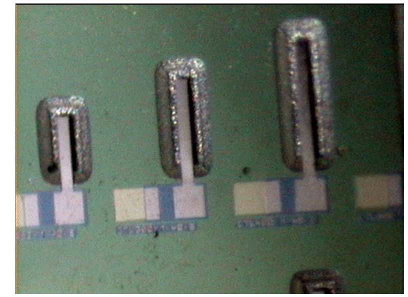
Transduction mechanisms

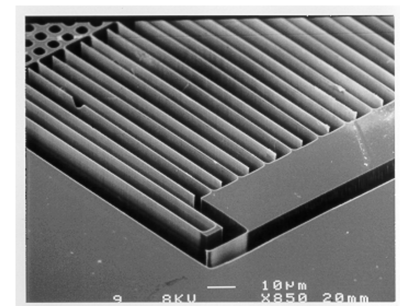
Piezoelectric: dynamical strain is converted into voltage difference.

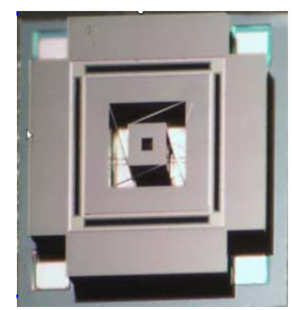
Capacitive: geometrical variations induce voltage difference

Inductive: dynamical oscillations of magnets induce electric current in coils









Transduction mechanisms 2

We focus on **Piezoelectric**: dynamical strain is converted into voltage difference.

Type	Governing Equation	Practical Maximum	Theoretical Maximum
Piezoelectric	$u = \sigma_y^2 k^2 / 2Y$	17.7 mJ/cm^3	335 mJ/cm^3
Electrostatic	$u = \frac{1}{2}\varepsilon E^2$	4 mJ/cm^3	44 mJ/cm^3
Electromagnetic	$u = B^2 / 2\mu_0$	4 mJ/cm^3	400 mJ/cm^3

Electrostatic is more easily scaled but you have to pay a debt: needs a bias voltage

Electromagnetic is more difficult to be scaled down

Theoretical model for the stochastic nonlinear piezoelectric cantilevers

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - K_V V + \zeta_z \\[2ex] \dot{V} = K_c\dot{x} - \dfrac{1}{\tau_p}V \end{cases}$$

with:

$$K_v = \frac{K_{eff} d_{31} a}{2t_p k_1}$$

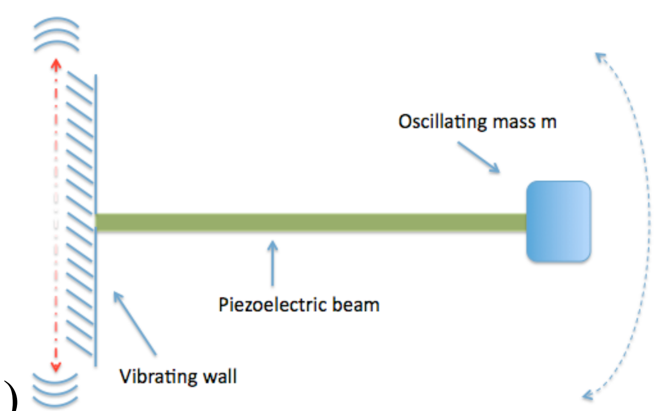
$$K_c = \frac{t_p d_{31} Y_p^E k_1}{a\varepsilon_p}$$

$$\tau_p = R_L C$$

The Physics of piezo materials applied to a cantilever

ζ_z Represents the stochastic vibration (force)

Thermal noise
Acoustic noise
Seismic noise
Ambient noise (wind, pressure fluctuations, …)
Man made vibrations (human motion, machine vibrations,…)



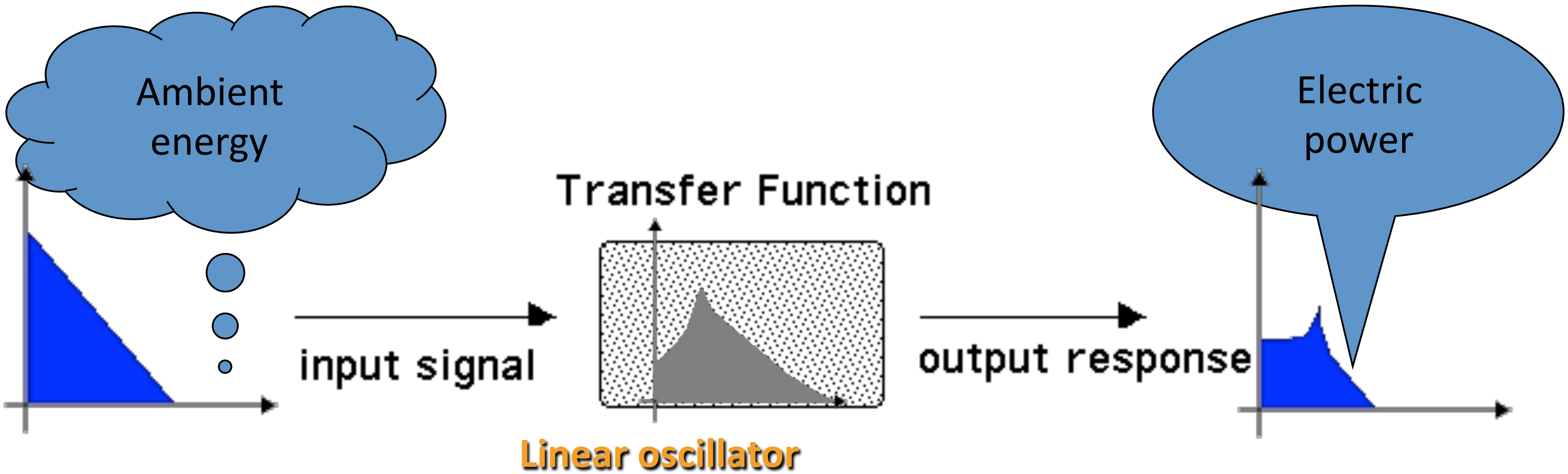
Linear system

If a linear system is considered: $U(x) \approx x^2$

1) There exist a simple math theory to solve the equations
2) They have a resonant behaviour (resonance frequency)
3) They can be "easily" realized with cantilevers and pendula

That's why people like them most!!!

Linear system 2

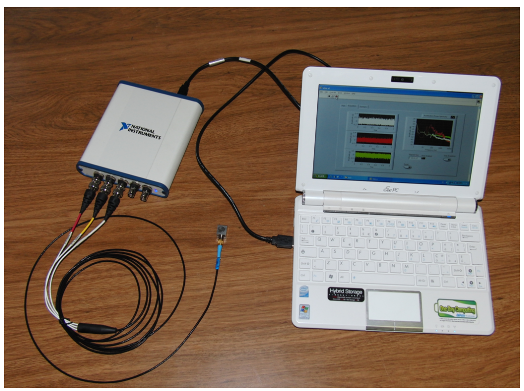


A linear system is the most performant if its resonace frequency is where the incoming energy is abundant…

This is a serious limitation when you want to build a small energy harvesting system…

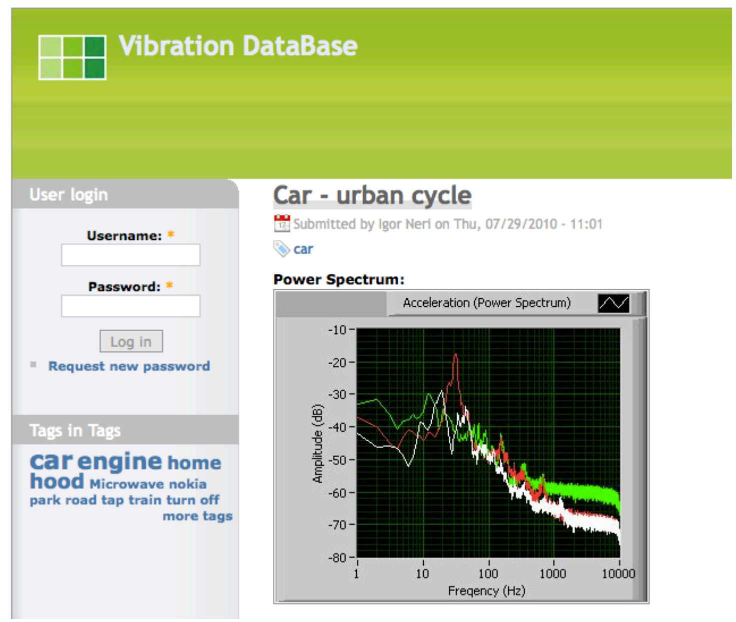
For two main reasons:

(1) the frequency spectrum of available vibrations instead of being sharply peaked at some frequency is usually very broad.
(as seen in F. Orfei's presentation).



Accelerometer:
- Tri axial
- Bandwidth from 0.4Hz to 10kHz
- ±50g

DAQ:
- 102.4 kS/s five simultaneous channel
- 4 channels with software-selectable IEPE signal conditioning
- USB powered

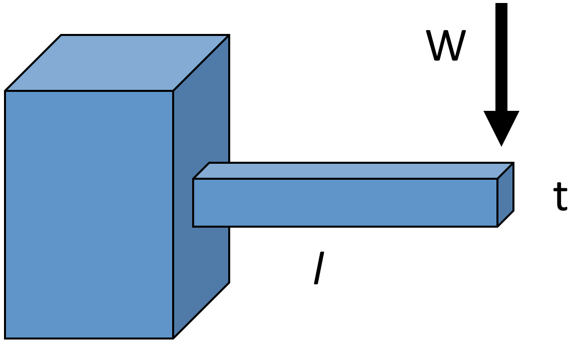


Signal presentation:
- Description
- Power spectrum
- Statistical data
- Time series download (only for authorized users)

(2) The frequency spectrum of available vibrations is particularly rich in energy in the low frequency part… and it is very difficult, if not impossible, to build small low-frequency resonant systems…

Resonant frequency ~ $[\mathrm{s}^{-1}]$

- MEMS cantilever 100 × 3 × 0.1 μm³, f_0=12 kHz
- NEMS cantilever 0.1 × 0.01 × 0.01 μm³, f_0=1.2 GHz



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \delta = \frac{Wl^3}{3EI} \qquad k = \frac{W}{\delta} = \frac{3EI}{l^3}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{3EI}{Ml^3}} = \frac{1}{2\pi}\sqrt{\frac{Ewt^3}{4Ml^3}} = \frac{t}{4\pi l^2}\sqrt{\frac{E}{\rho}}$$

Let's look an example…

Description of the resonator design

The resonator design is a square shaped block of single crystal silicon with dimensions of 320x320x28 um^3 (design H1). Its main resonance mode is the so called square extensional (SE) resonance, which is characterized by its zoom-in/zoom-out oscillation. The resonance is excited by a piezoelectric AlN thin film on top of the resonator block. The electrically conductive (p-doped) silicon block acts as the bottom electrode, and a molybdenum thin film has been patterned to provide the top electrode. See reference [1] for a general description of the SE resonator. Reference [2] discusses piezoelectric excitation of the SE resonance mode.

Figure 3 shows how the resonator is recommended to be connected.

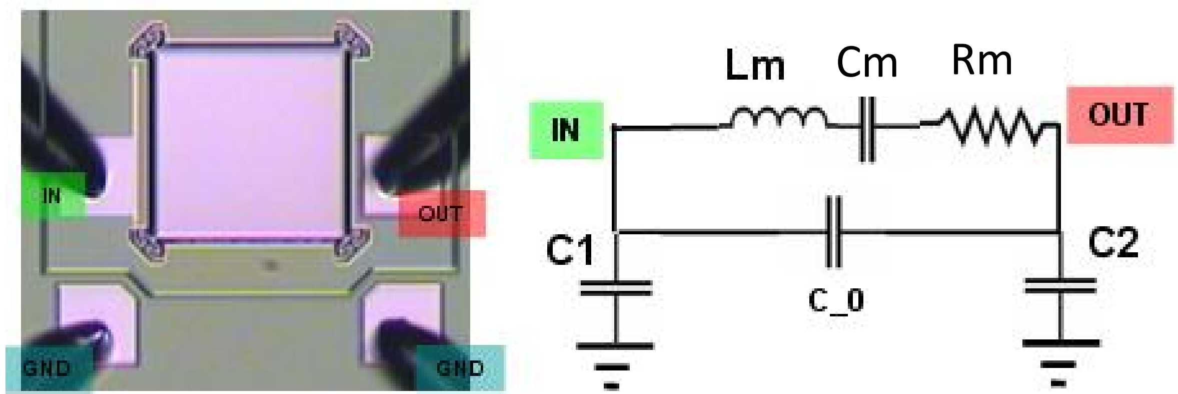
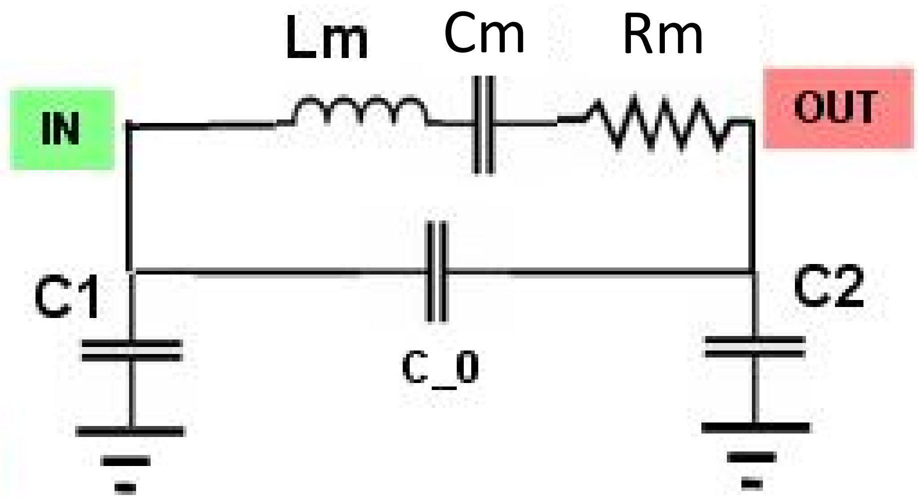


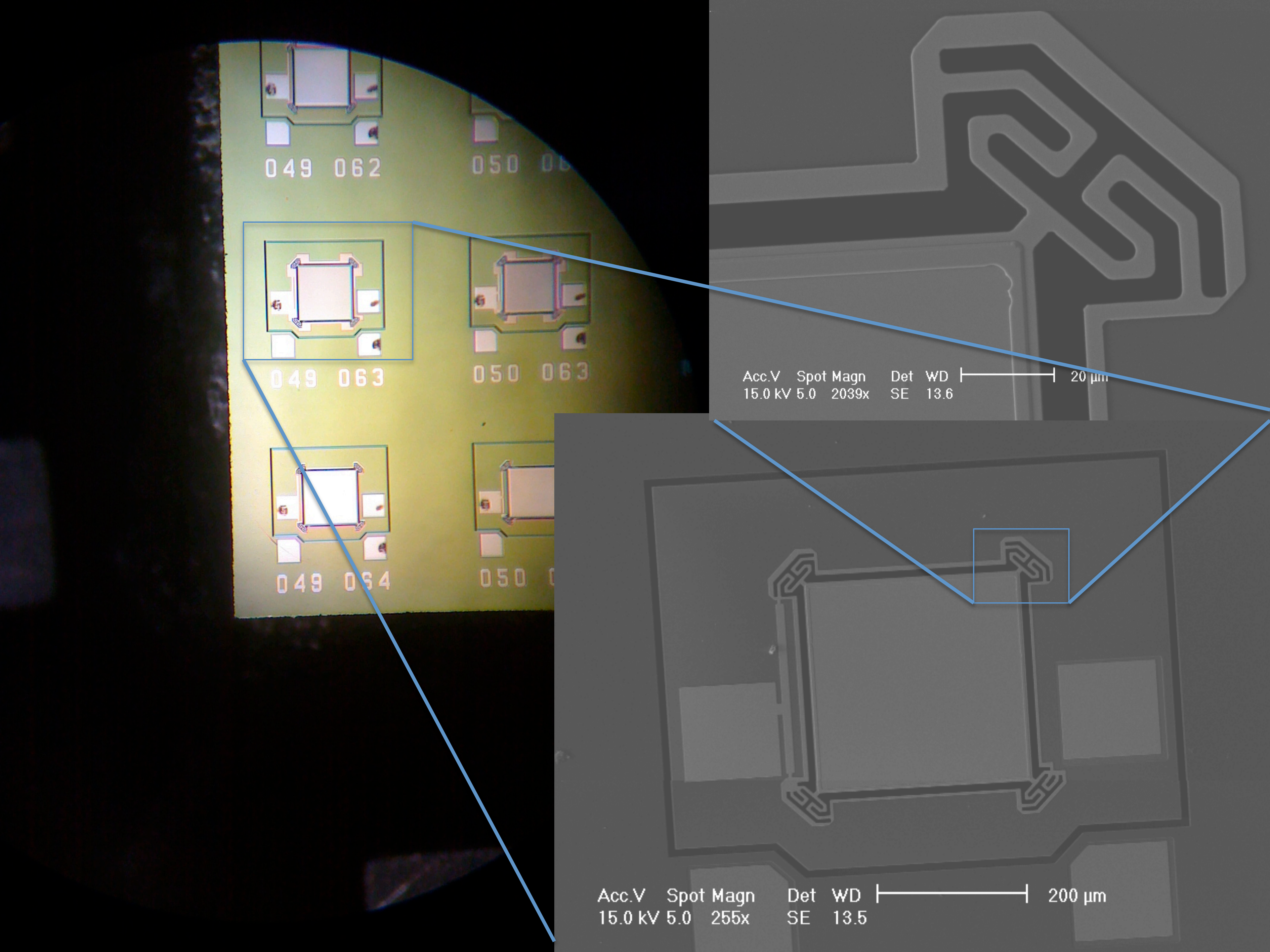
Figure 3: Electrical connection of the resonator.

References
[1] V. Kaajakari et al., “Square-Extensional Mode Single-Crystal Silicon Micromechanical Resonator for Low-Phase-Noise Oscillator Applications,” *IEEE Electron Device Letters* 25, no. 4 (4, 2004): 173-175.
[2] A. Jaakkola et al., “Piezoelectrically transduced Single-Crystal-Silicon Plate Resonators,” in *IEEE Ultrasonics Symposium* (presented at the IEEE Ultrasonics Symposium, Beijing, China, 2008), 2181 – 2184



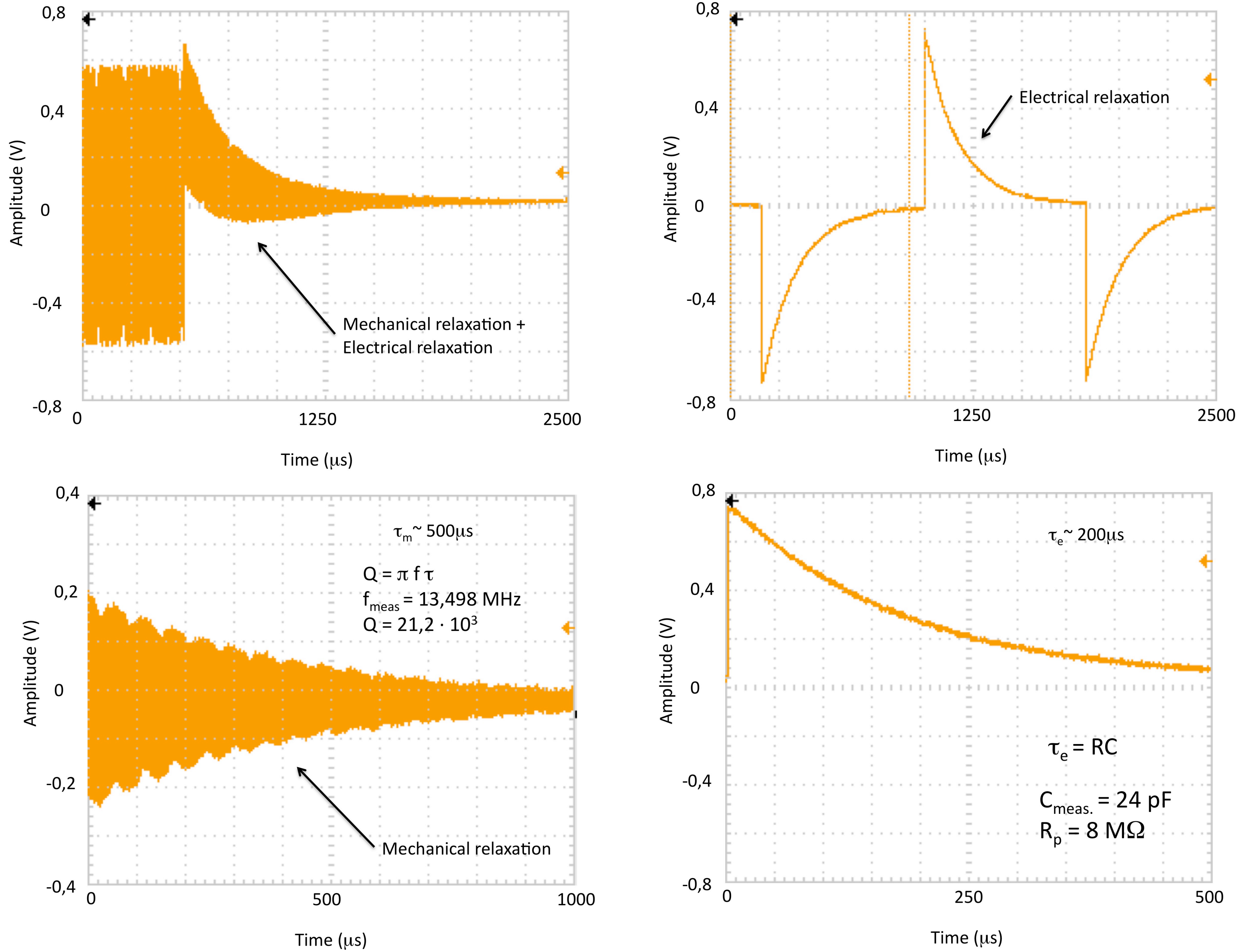
parameter	value	unit
Rm	128.9606	Ohm
Cm	1.01E-14	F
Lm	0.013691	H
C0	3.00E-11	F
f0	13.5	MHz
Q	9000	1
k2eff	0.04	%
C1	<1e-12	F
C2	<1e-12	F

Table 1: equivalent circuit parameters (and their derivatives).



049 062
050 06
049 063
050 063
049 064
050 0
Acc.V Spot Magn Det WD 20 µm
15.0 kV 5.0 2039x SE 13.6
Acc.V Spot Magn Det WD 200 µm
15.0 kV 5.0 255x SE 13.5

VTT Membrane characterization



From the model of a linear oscillator:

The voltage transfer function is:

$$|H(\omega)| = \frac{1}{m}\frac{\omega k_c}{\sqrt{\omega^2\left(\frac{k+\frac{\gamma}{\tau}+k_c k_v}{m} - \omega^2\right)^2 + \left(\left(\frac{\gamma}{m}+\frac{1}{\tau}\right)\omega^2 - \frac{k}{m\tau}\right)^2}}$$

or considering: $\omega_0 = \sqrt{\frac{k+\frac{\gamma}{\tau}+k_c k_v}{m}}$

and: $\omega_1 = \sqrt{\frac{k}{\gamma\tau + m}}$

$$\Rightarrow \quad |H(\omega)| = \frac{1}{m}\frac{\omega k_c}{\sqrt{\omega^2\left(\omega^2-\omega_0^2\right)^2 + \left(\frac{\gamma\tau+m}{m\tau}\right)^2\left(\omega^2-\omega_1^2\right)^2}}$$

if $\omega^2\left(\omega^2-\omega_0^2\right)^2 > \left(\frac{\gamma\tau+m}{m\tau}\right)^2\left(\omega^2-\omega_1^2\right)^2$

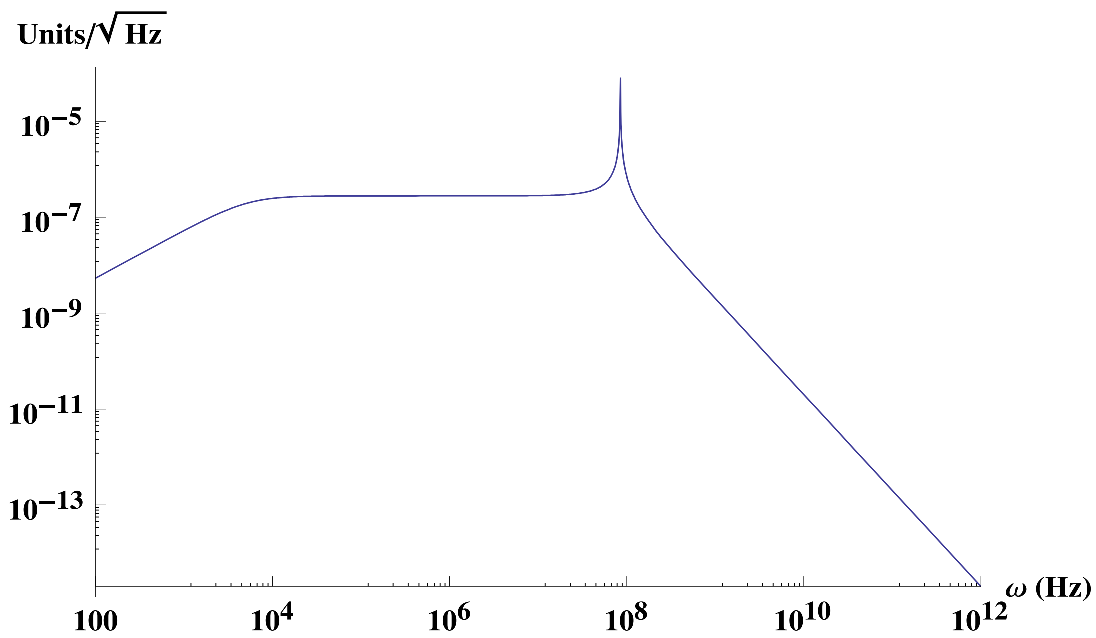
the resonance frequency is $\omega_0 = \sqrt{\frac{k+\frac{\gamma}{\tau}+k_c k_v}{m}}$

where $|H(\omega)|_{max} = \frac{\omega_0 k_c \tau}{(\gamma\tau+m)\left(\omega_0^2-\omega_1^2\right)^2}$

if $\omega^2\left(\omega^2-\omega_0^2\right)^2 < \left(\frac{\gamma\tau+m}{m\tau}\right)^2\left(\omega^2-\omega_1^2\right)^2$

the resonance frequency is $\omega_1 = \sqrt{\frac{k}{\gamma\tau+m}}$

where $|H(\omega)|_{max} = \frac{k_c}{m\left|\omega_1^2-\omega_0^2\right|}$



The analytic result for the Q is:

$$Q = \frac{\omega_r}{\Delta\omega}$$

ω_r is the resonance frequency and $\Delta\omega$ is the bandwidth (full width when the output voltage is $V_{max}/\sqrt{2}$)

$$
\begin{aligned}
\text{Quality Factor} = \Big\{ \Big(& 3\,k\,m\,\tau^2 \sqrt{\Big(2\,m^2 + 2\,(k + kc\,kv)\,m\,\tau^2 - \gamma^2\,\tau^2 +} \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big)\Big) \Big/ \Big(\sqrt{\Big(2\,m^6 + 6\,(k} \\
& - 2\,kc\,kv)\,m^5\,\tau^2 + 2\,\gamma^6\,\tau^6 - 2\,\gamma^4\,\tau^4 \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)} + 4\,(k + kc\,kv)\,m\,\gamma^2 \\
& \tau^4\,\Big(-3\,\gamma^2\,\tau^2 + 2\,\sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big) - \\
& m^4\,\Big(3\,\gamma^2\,\tau^2 + 18\,kc\,kv\,\gamma\,\tau^3 - 6\,k^2\,\tau^4 + 6\,k\,kc\,kv\,\tau^4 - 15\,kc^2\,kv^2\,\tau^4 + 2 \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big) + m^2\,\tau^2\,\Big(-3\,\gamma^4 \\
& \tau^2 - 18\,kc\,kv\,\gamma^3\,\tau^3 + 12\,kc\,kv\,\gamma\,\tau \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)} - 2\,(k + kc\,kv)^2 \\
& \tau^2\,\sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)} + \gamma^2\,\Big(15\,k^2\,\tau^4 \\
& + 30\,k\,kc\,kv\,\tau^4 + 15\,kc^2\,kv^2\,\tau^4 + 2 \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big)\Big) + 2\,m^3\,\tau^2\,\Big(\\
& k^3\,\tau^4 + 3\,k^2\,kc\,kv\,\tau^4 + k\,\Big(6\,\gamma^2\,\tau^2 + 18\,kc\,kv\,\gamma\,\tau^3 + 3\,kc^2\,kv^2\,\tau^4 - 2 \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big) + kc\,kv\,\Big(-3 \\
& \gamma^2\,\tau^2 + 18\,kc\,kv\,\gamma\,\tau^3 + kc^2\,kv^2\,\tau^4 + 4 \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big)\Big)\Big)}\Big)\Big\}
\end{aligned}
$$

Whish list for the perfect vibration harvester

1) Capable of harvesting energy on a broad-band
2) No need for frequency tuning
3) Capable of harvesting energy at low frequency

1) Non-resonant system
2) “Transfer function” with wide frequency resp.
3) Low frequency operated

PRL 102, 080601 (2009) PHYSICAL REVIEW LETTERS week ending 27 FEBRUARY 2009

Nonlinear Energy Harvesting

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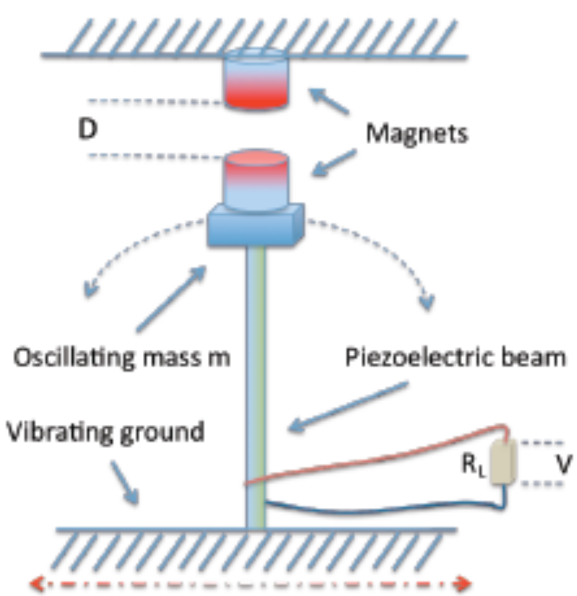
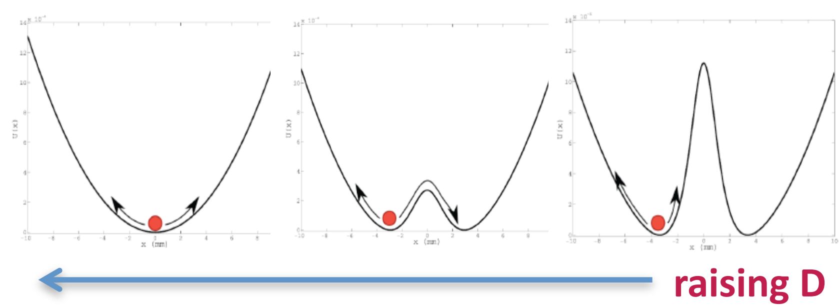
(Received 18 September 2008; published 23 February 2009)

Ambient energy harvesting has been in recent years the recurring object of a number of research efforts aimed at providing an autonomous solution to the powering of small-scale electronic mobile devices. Among the different solutions, vibration energy harvesting has played a major role due to the almost universal presence of mechanical vibrations. Here we propose a new method based on the exploitation of the dynamical features of stochastic nonlinear oscillators. Such a method is shown to outperform standard linear oscillators and to overcome some of the most severe limitations of present approaches. We demonstrate the superior performances of this method by applying it to piezoelectric energy harvesting from ambient vibration.

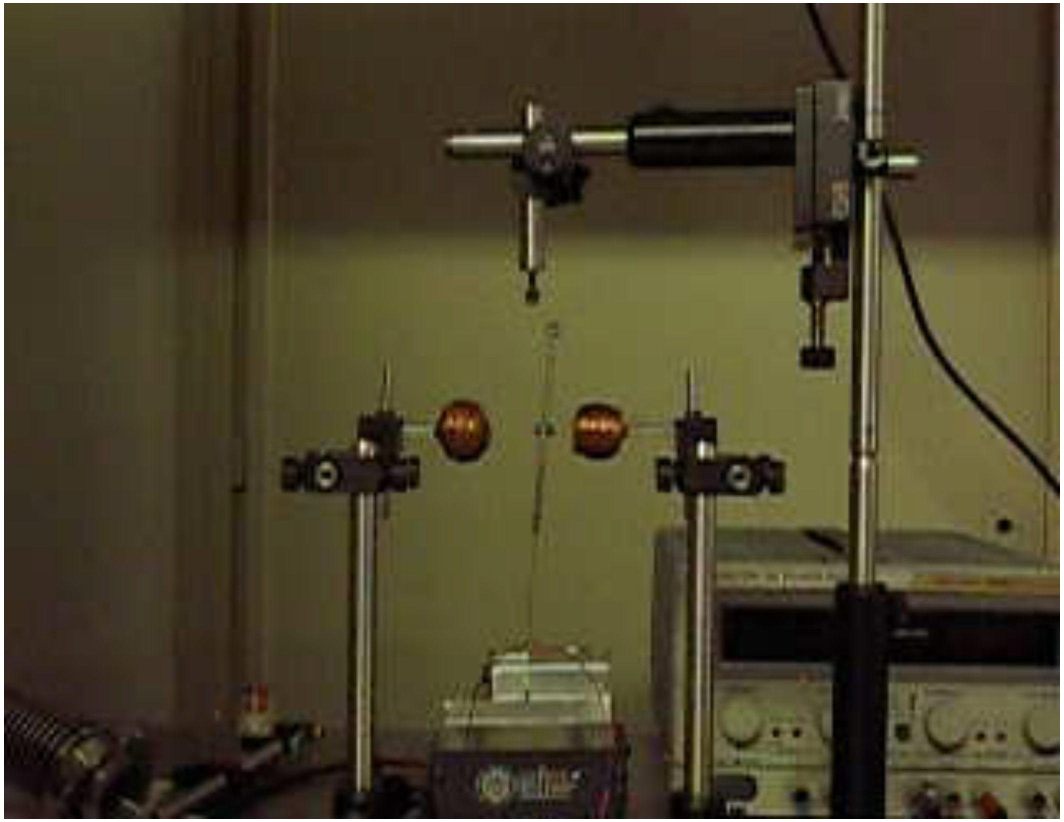
DOI: 10.1103/PhysRevLett.102.080601 PACS numbers: 05.40.Ca, 05.10.Ln, 05.45.−a, 84.60.−h





Result: output power is maximum for in an optimal nonlinear regime

NON-Linear mechanical oscillators

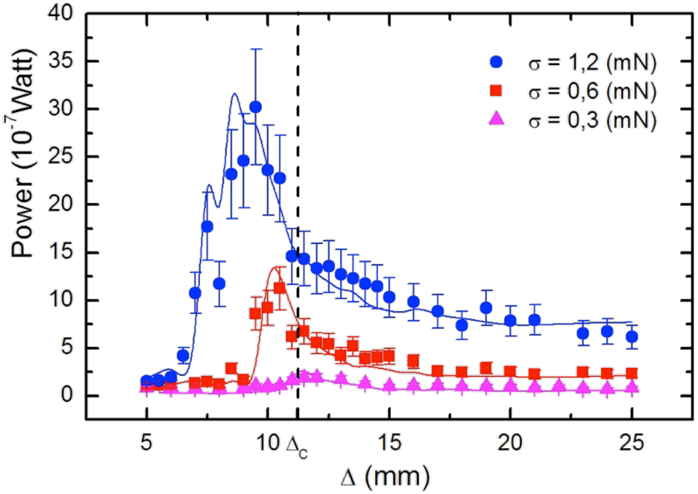


http://www.nipslab.org/node/1676

U(x) was varied by having recourse to an inverted pendulum, so that

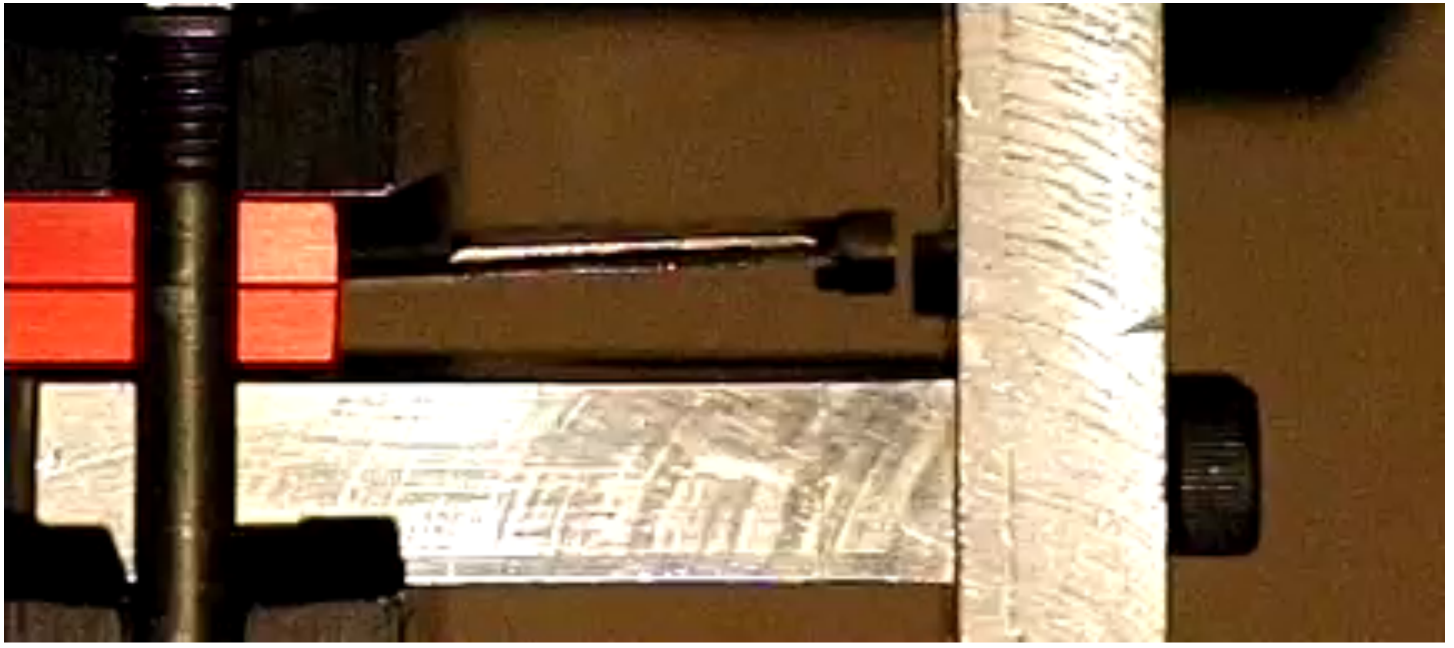
$$U(x) = \frac{1}{2}k_e x^2 + (Ax^2 + BD^2)^{-\frac{3}{2}}$$

Harvester output power: **$W = V^2/R_L$**



NON-Linear mechanical oscillators 2

Slow motion movie: 200 Frame/s

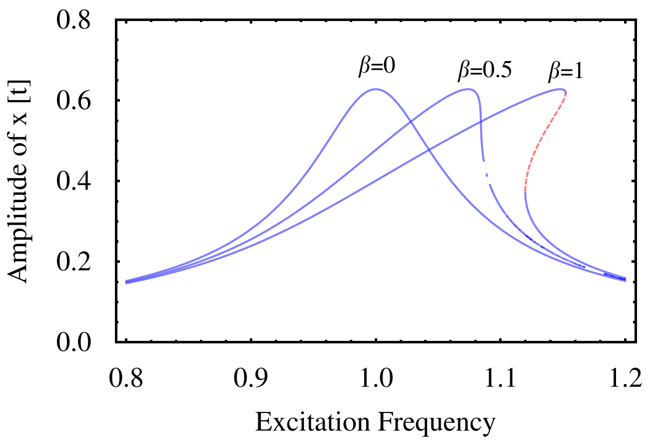


Only bistability???

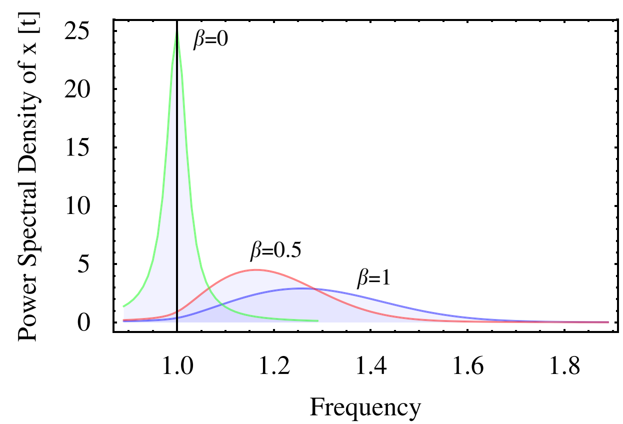
Considering a Duffing oscillator: $\ddot{x} + \gamma\dot{x} + x + \beta x^3 = F(t)$

γ is the effective damping ratio for both electrical and mechanical damping

$\beta > 0$ is a stiffness nonlinearity coefficient



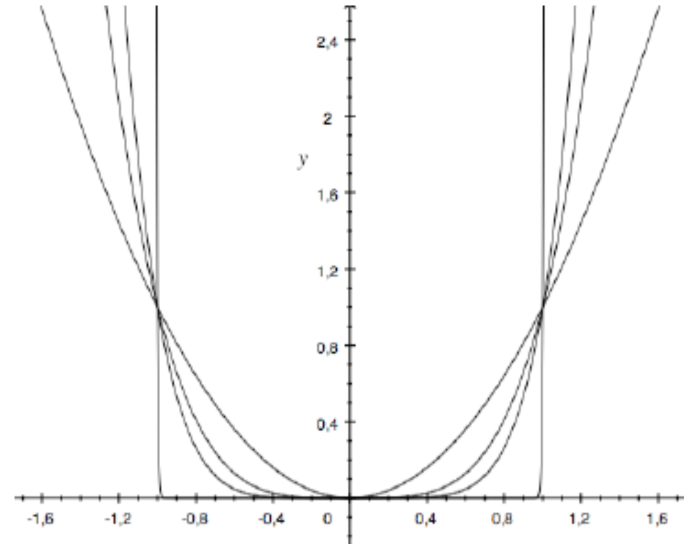
Steady-state frequency response curves under harmonic excitations of a fixed frequency: Non Linear Resonance

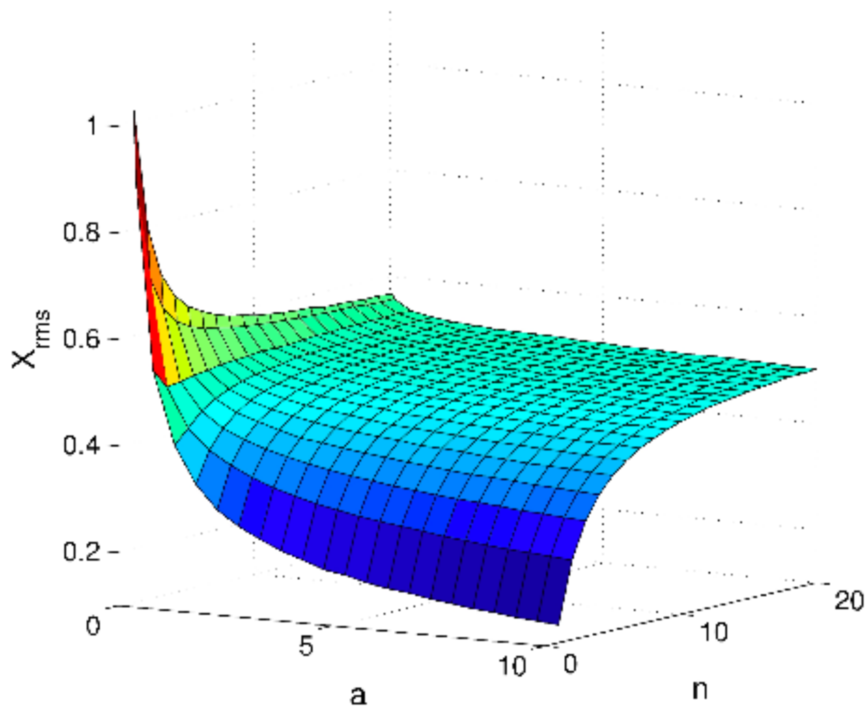


Power spectral density curves of *x(t)* under White Gaussian excitations of a fixed spectral density.

A general monostable potential

$$U(x) = ax^{2n} \quad \text{with} \quad \begin{array}{l} a > 0 \\ n = 1,2,\ldots \end{array}$$





Numerical simulation with an exponentially correlated noise with correlation time τ:

$$\left\langle \xi(t)\xi(t_1)\right\rangle = \sigma^2 e^{-\frac{|t-t_1|}{\tau}}$$

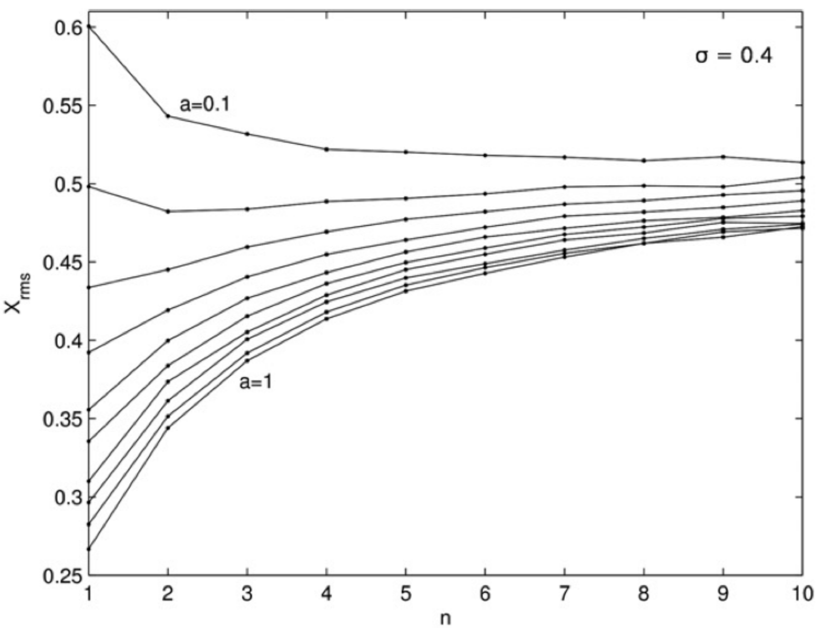
There exists a threshold amplitude a_{th}:

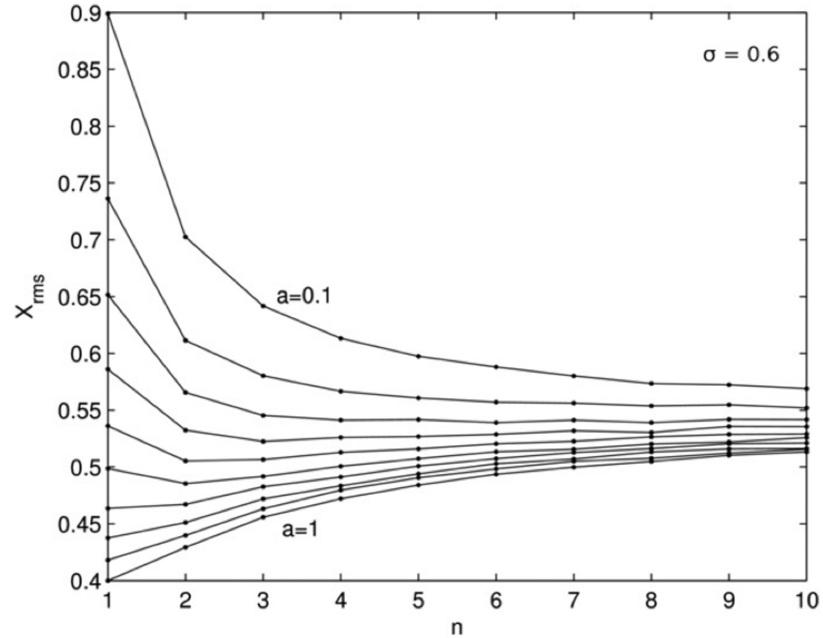
$$a_{th} \approx \frac{D}{4} = \sigma^2\tau$$

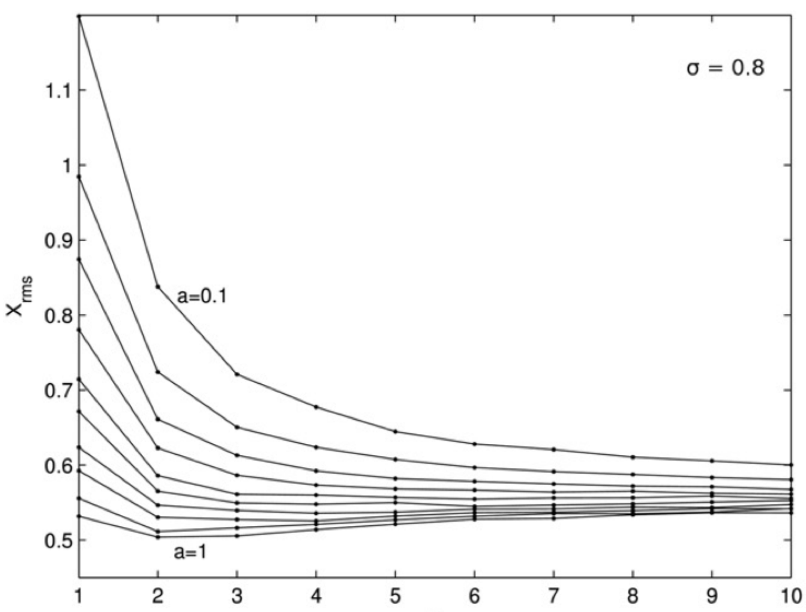
Above which the nonlinear system outperforms the linear one.

L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

Varying the noise amplitude

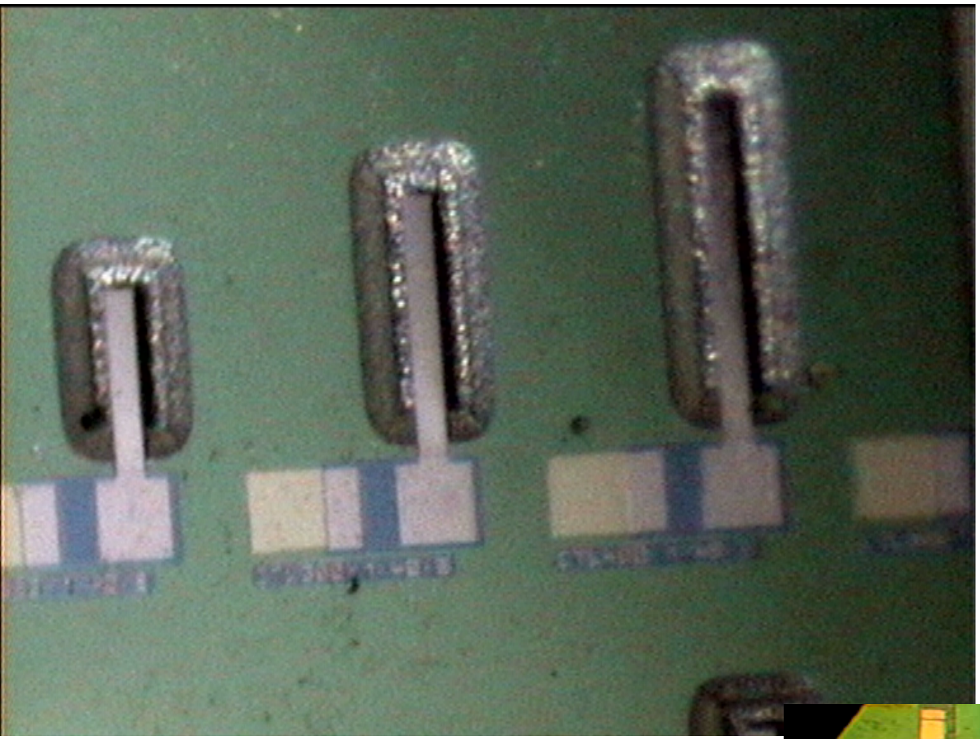






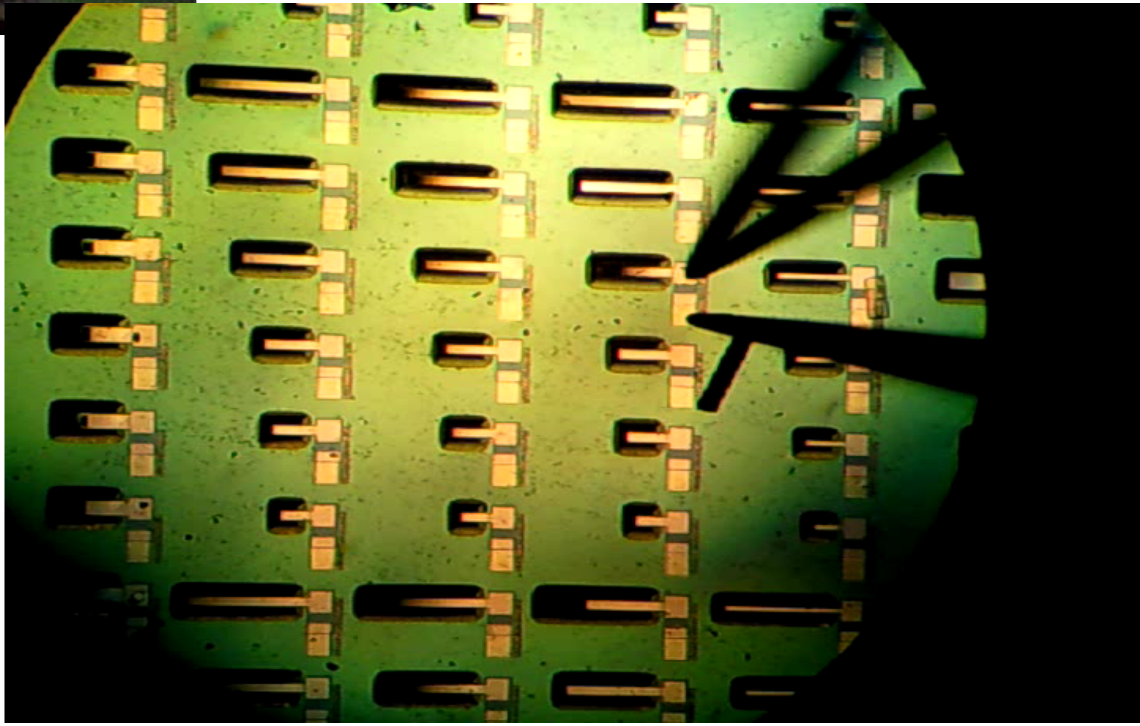
Once σ and a are fixed the choice of a linear (n = 1) or nonlinear potential (n ≥ 2) can be made in order to maximize x_{rms} and consequently the power obtained at the device output.

L. Gammaitoni I Neri and H Vocca, Chem. Phys. (2010), doi:10.1016/j.chemphys.2010.08.012

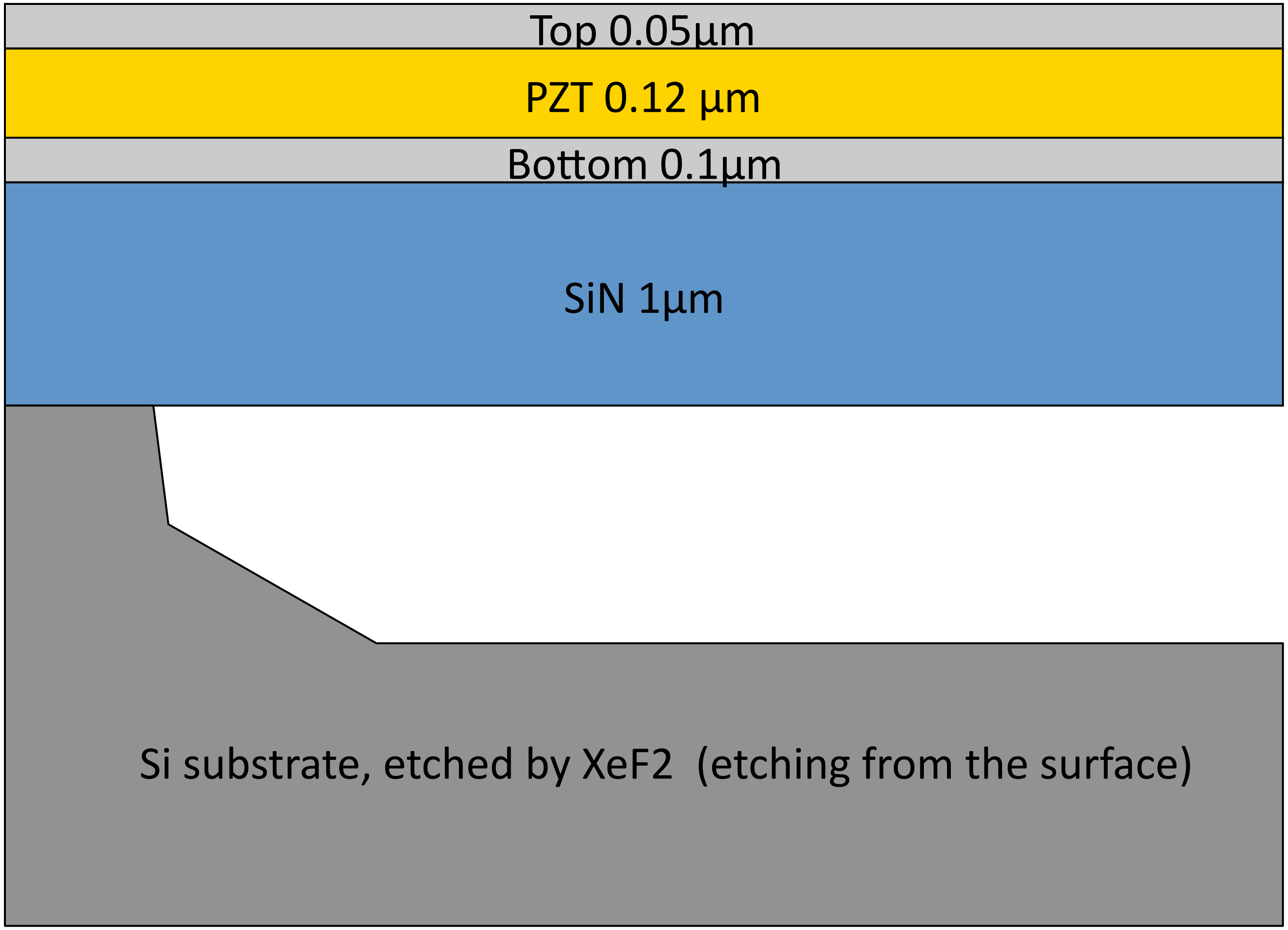


In collaboration with CEA-Leti we are investigating μcantilevers dynamics

some are nonlinear



Scheme of the cross section of one cantilever



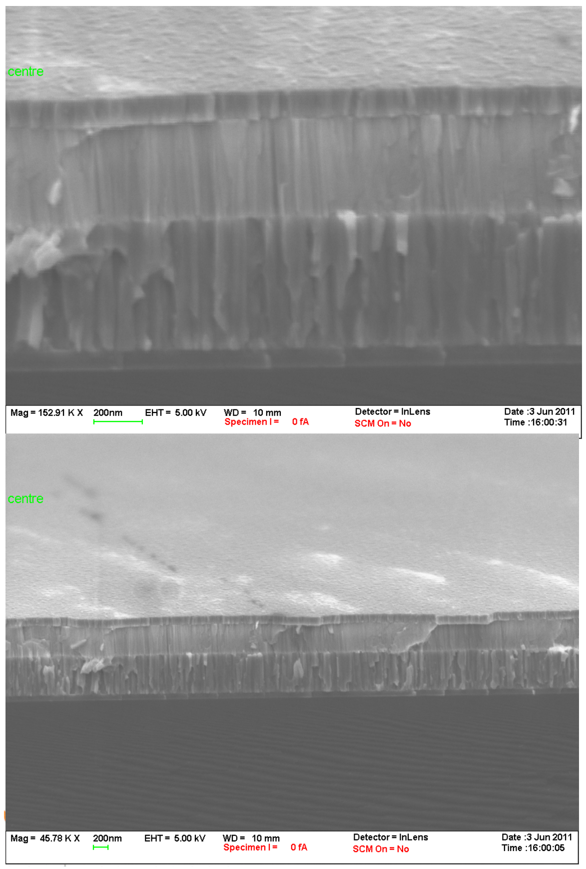
Typical electrical features

- Max voltage sustainable: 5-7V
- e31 PZT = $-5 C/m^2$
- Dielectric constant PZT: 1000

New (nonlinear) membranes from VTT

Free-standing AlO/Mo/AlN/Mo membranes

- Good overall quality across the wafer, low roughness, no defects

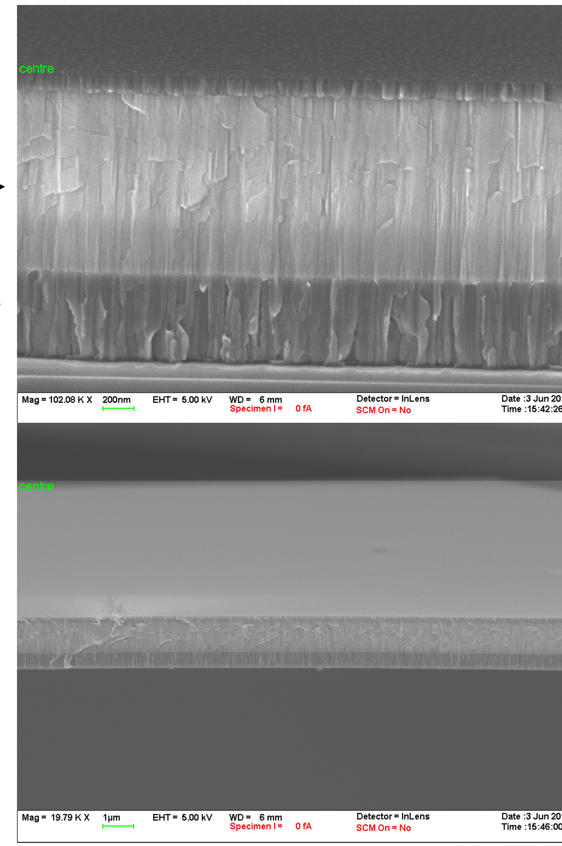


Mo 100nm

AlN 300/1000nm

Mo 500nm

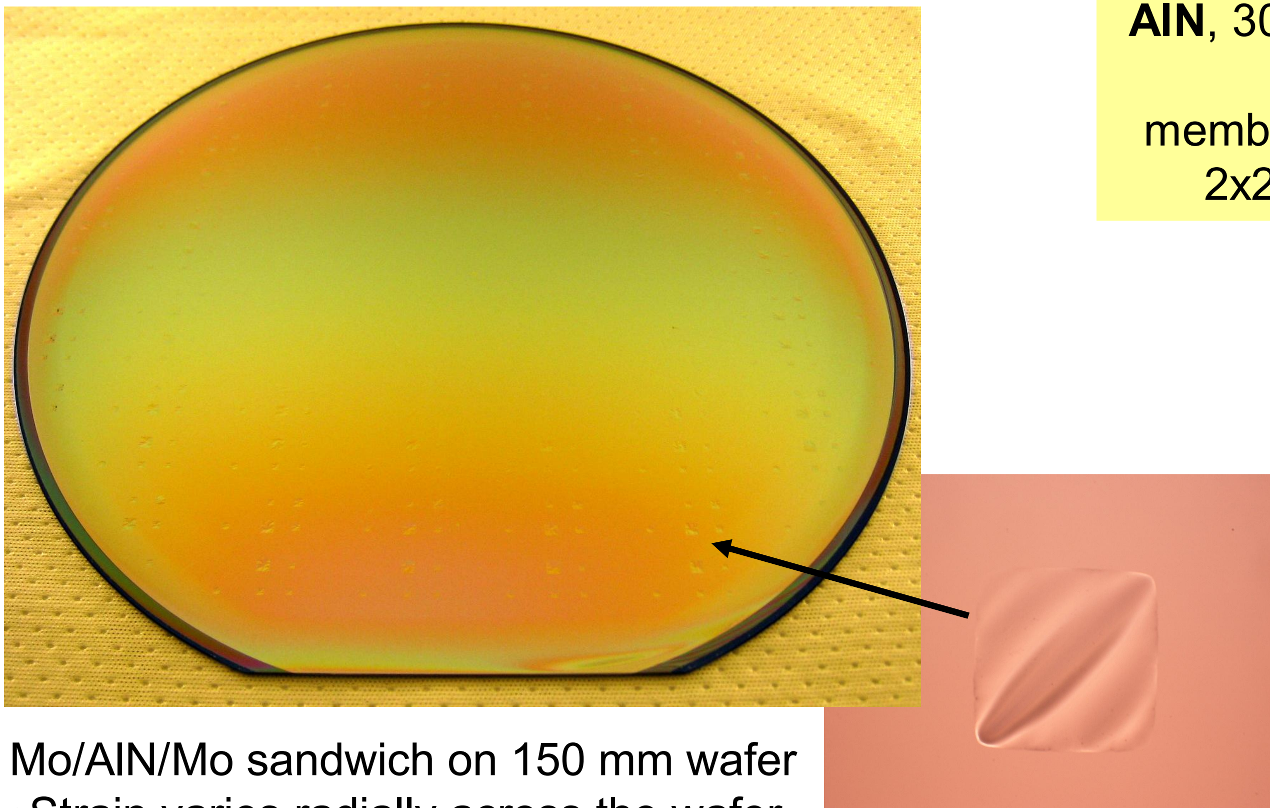
AlO 50nm



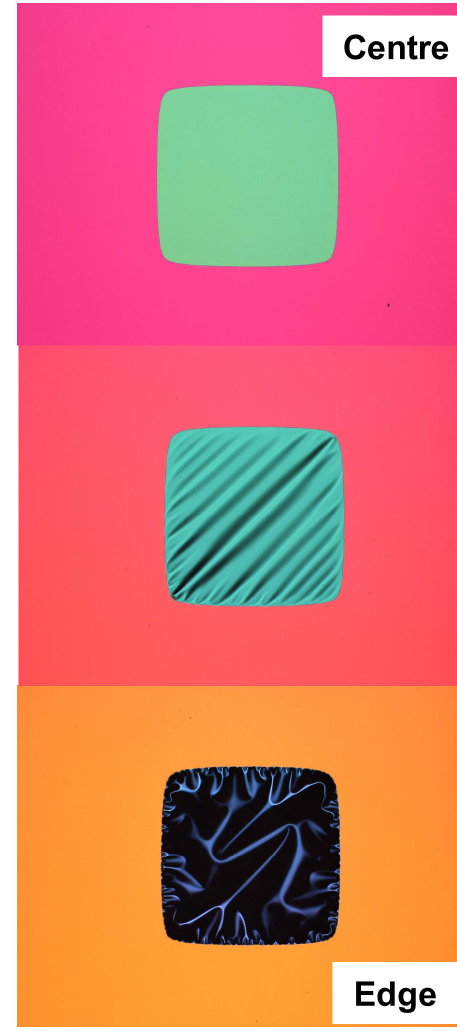
Cross-sectional SEM images of free-standing membranes

New (nonlinear) membranes from VTT

AlN, 300 nm thick membranes 2x2 mm^2



Mo/AlN/Mo sandwich on 150 mm wafer
- Strain varies radially across the wafer



NANOPOWER

Conclusions

1) Non linear mechanical oscillators outperform linear ones in noisy environments;

2) Non-linear systems are more difficult to treat but more interesting…

3) Bistability is not the only nonlinearity available;

4) The same principles are also valid for capacitive and inductive harvesters;

5) A great amount of work has still to be done especially at small scales!!!