Noise-induced currents in open quantum dots

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26.10.2011

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Outline

Introduction

- Noise-induced currents in quantum dots
- Motivation

Model & Technique

- Model
- Semiclassical approach

Results

- I-V characteristics of single cavity
- Current induced by hot resistor
- Double cavity: Current from hot spots

Summary

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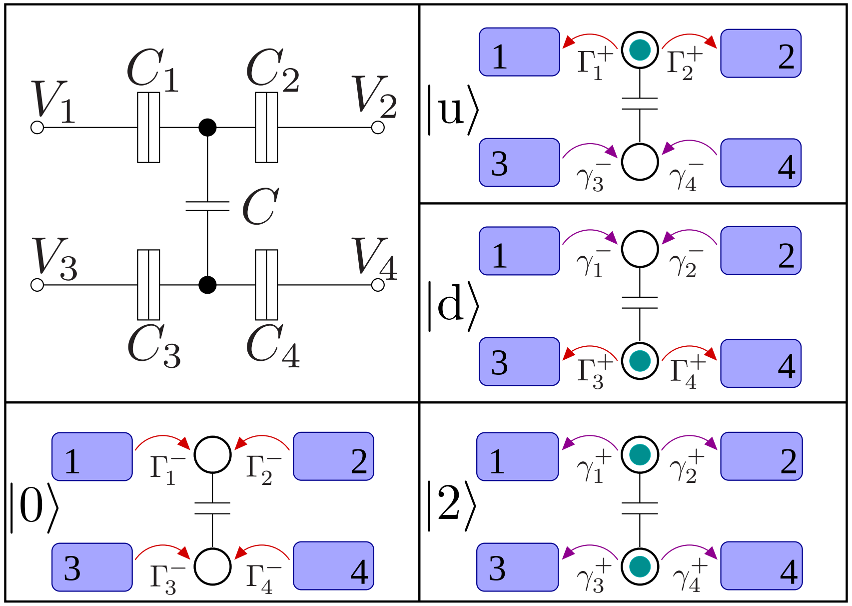
Introduction

Noise-induced currents in quantum dots

Sánchez, López, Sánchez, Büttiker, PRL 2010



- Double quantum dot with four contacts
- Coulomb blockade regime
- Energy-dependent, asymmetric tunneling rates: $\Gamma_1^+\Gamma_2^- \neq \Gamma_1^-\Gamma_2^+$
- Nonequilibrium noise of driven dot induces current through undriven dot: Coulomb drag/Photo-induced current

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Noise-induced currents in quantum dots

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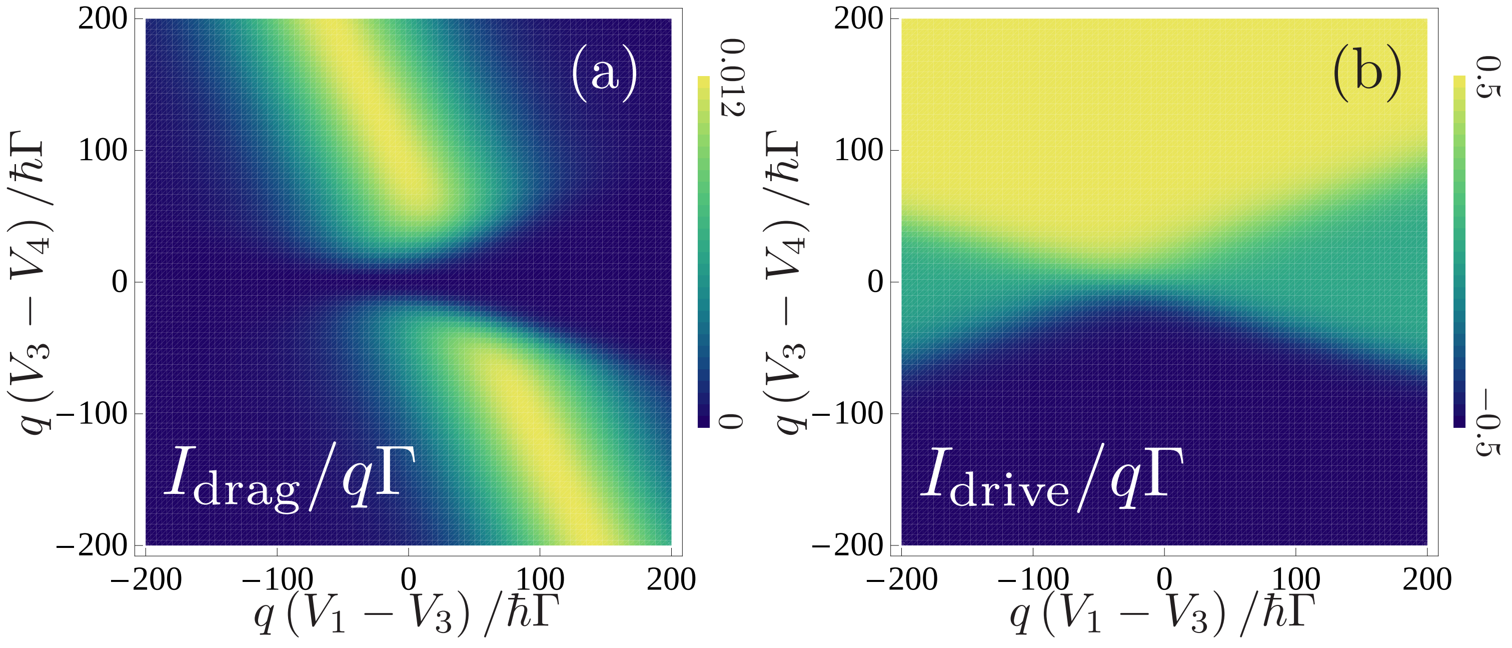

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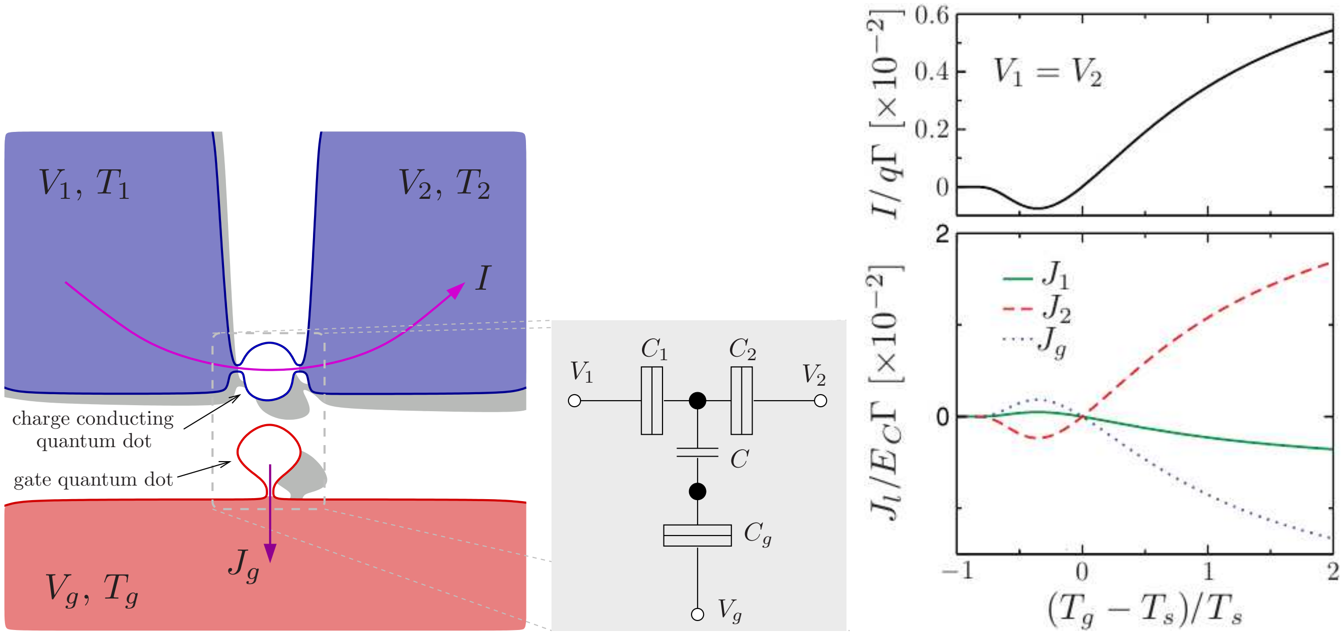
Noise-induced currents in quantum dots

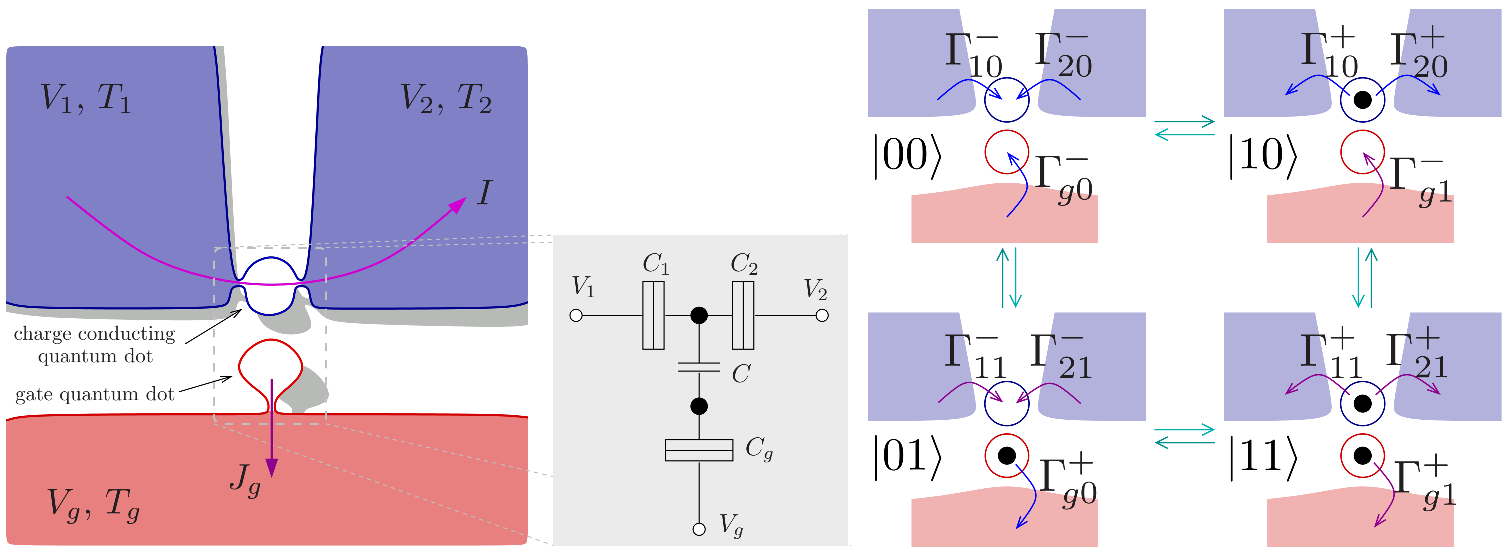
Sánchez, Büttiker, PRB 2011



- Three-terminal setup
- Thermal fluctuations of gate dot occupancy
- Optimal heat to charge conversion
- One energy quantum of the bath transfers one charge quantum

Noise-induced currents in quantum dots

Sánchez, Büttiker, PRB 2011



- Three-terminal setup
- Thermal fluctuations of gate dot occupancy
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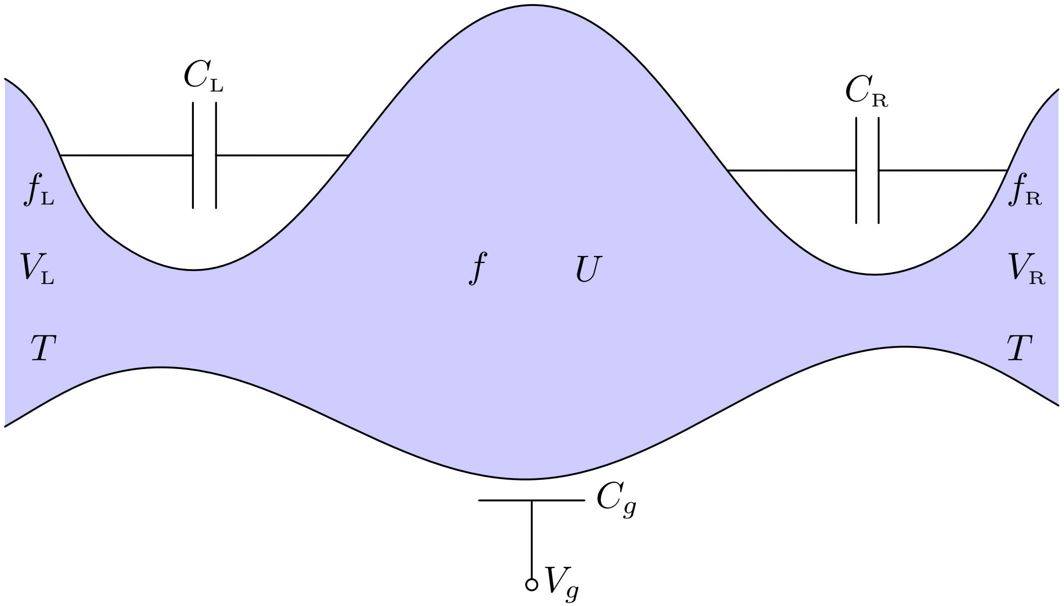
Motivation

Consider transport through open quantum dots!

- Very small currents in Coulomb-blockade system
- Not optimal for applications
- How do effects scale with the system size?

Model & Technique

Model



- Cavity coupled to 2 leads via quantum point contacts
- Chaotic cavity subject to potential fluctuations δU
- Energy-dependent transmissions $T_r = T_r^0 - eT_r'\delta U$
- Large channel number $N = \frac{\text{channel width}}{\lambda_{\text{Fermi}}} \Rightarrow$ Semiclassical approach

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Theoretical description

- Kinetic equation for cavity distribution function $f = \sum_r \frac{T_r}{T_\Sigma} f_r + \delta f$

$$e\nu_{\mathsf{F}} \frac{df}{dt} = e\nu_{\mathsf{F}} \frac{\partial f}{\partial U} \dot{U} + \frac{e}{h} \sum_r T_r (f_r - f) + \delta i_\Sigma$$

- Write charge in cavity via f and via C_i, V_i, U

  $\Rightarrow$ Relation between fluctuations δf and δU

$$\int dE \delta f = e \left(\frac{C_\Sigma}{C_\mu} + \frac{\chi}{e^2 \nu_{\mathsf{F}}} \right) \delta U + \frac{\chi}{\nu_{\mathsf{F}}} \frac{T'_\Sigma}{T^0_\Sigma} (\delta U)^2$$

  where $\chi = e^3 \nu_{\mathsf{F}} \frac{T'_r T^0_{\bar{r}} - T'_{\bar{r}} T^0_r}{(T^0_\Sigma)^2} (V_r - V_{\bar{r}})$

- Eliminate δf from kinetic equation

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Theoretical description

Nonlinear Langevin equation determining the potential fluctuations δU

$$C_\Sigma \delta \dot{U} = -\frac{e^2}{h} T_\Sigma^0 \left(\frac{C_\Sigma}{C_\mu} + \frac{\chi}{e^2 \nu_\mathsf{F}} \right) \delta U + \frac{e^3}{h} T_\Sigma' \frac{C_\Sigma}{C_\mu} (\delta U)^2 + \delta I_\Sigma$$

- Diffusion coefficients $\langle \delta I_r(t) \delta I_r(t') \rangle = D_r \delta(t - t')$ depend on δU!
- Noise term has to be interpreted according to Klimontovich prescription Klimontovich, Phys. A, 1990
- Convert Langevin equation into Fokker-Planck equation
- Determine fluctuations $\langle \delta U \rangle$, $\langle (\delta U)^2 \rangle$
- Calculate current $I_r = \frac{e}{h} \int dE T_r (f_r - f) + \delta I_r$

Results

Single cavity

Expand current in powers of applied voltage $V_r - V_{\bar{r}}$

$$\langle I_r \rangle = \langle I_r \rangle^{(1)} + \langle I_r \rangle^{(2)} + \cdots$$

$$\langle I_r \rangle^{(1)} = \frac{e^2}{h} \left[\frac{T_r^0 T_{\bar{r}}^0}{T_\Sigma^0} (V_r - V_{\bar{r}}) + e \frac{C_\Sigma}{C_\mu} \frac{T_r' T_{\bar{r}}^0 - T_{\bar{r}}' T_r^0}{T_\Sigma^0} \langle (\delta U)^2 \rangle^{(1)} \right]$$

- First term: Standard current through cavity
- Second term: Single-channel interaction correction over many-channel background, due to energy-dependent transmissions

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Single cavity

Expand current in powers of applied voltage $V_r - V_{\bar{r}}$

$$\langle I_r \rangle = \langle I_r \rangle^{(1)} + \langle I_r \rangle^{(2)} + \cdots$$

$$\langle I_r \rangle^{(2)} = -\frac{e^3}{h} \frac{T_r^0 T_{\bar{r}}'}{T_\Sigma^0} \langle \delta U \rangle^{(1)} (V_r - V_{\bar{r}}) - \frac{e^2}{h} \frac{C_\Sigma}{C_\mu} \left(T_r^0 \langle \delta U \rangle^{(2)} - e T_r' \langle (\delta U)^2 \rangle^{(2)} \right) + \frac{D_{1r}^{(2)} + 2 D_{2r}^{(1)} \langle \delta U \rangle^{(1)}}{2 C_\Sigma}$$

- Energy-dependent transmissions and fluctuating potential yield quadratic terms
- Potentially useful for rectification effects

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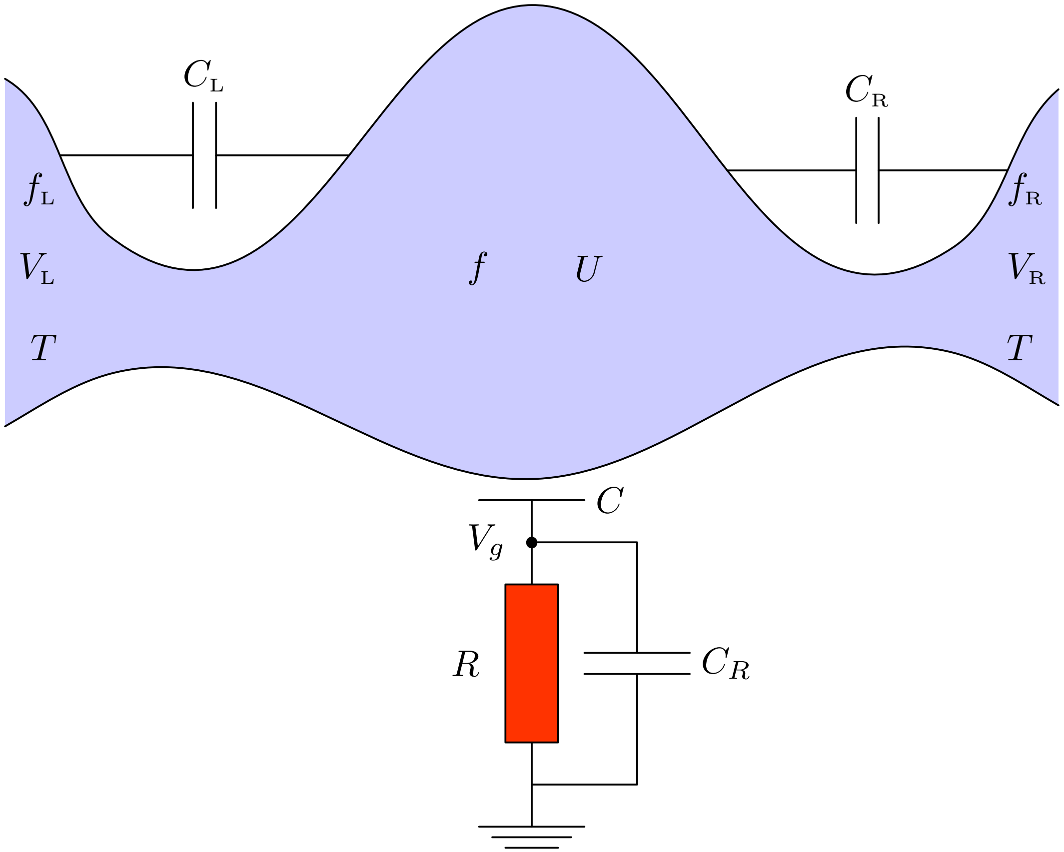
Current induced by hot resistor



- Couple resistor R capacitively to the cavity
- Resistor at temperature $T_R \neq T$
- Capacitance C_R across resistor for regularization in limit $C \to 0$

Current induced by hot resistor

Current through cavity:

For $G'_r = \frac{2e^3}{h} T'_r \ll G_r = \frac{e^2}{h} T^0_r$

$$\langle I_r \rangle = \frac{1}{C_{\text{eff}}} \frac{G_r G'_{\bar{r}} - G_{\bar{r}} G'_r}{G_\Sigma} k_{\mathsf{B}} (T - T_R)$$

- Effective capacitance describes coupling of cavity and resistor
- Second term captures left-right and particle-hole symmetry
- Current linear in temperature difference

Current induced by hot resistor: Efficiency

- Heat current from resistor into cavity

$$\langle J_E \rangle = \frac{1}{C_{\text{eff}}} G_\Sigma k_{\text{B}} (T - T_R)$$

- Maximal extracted power

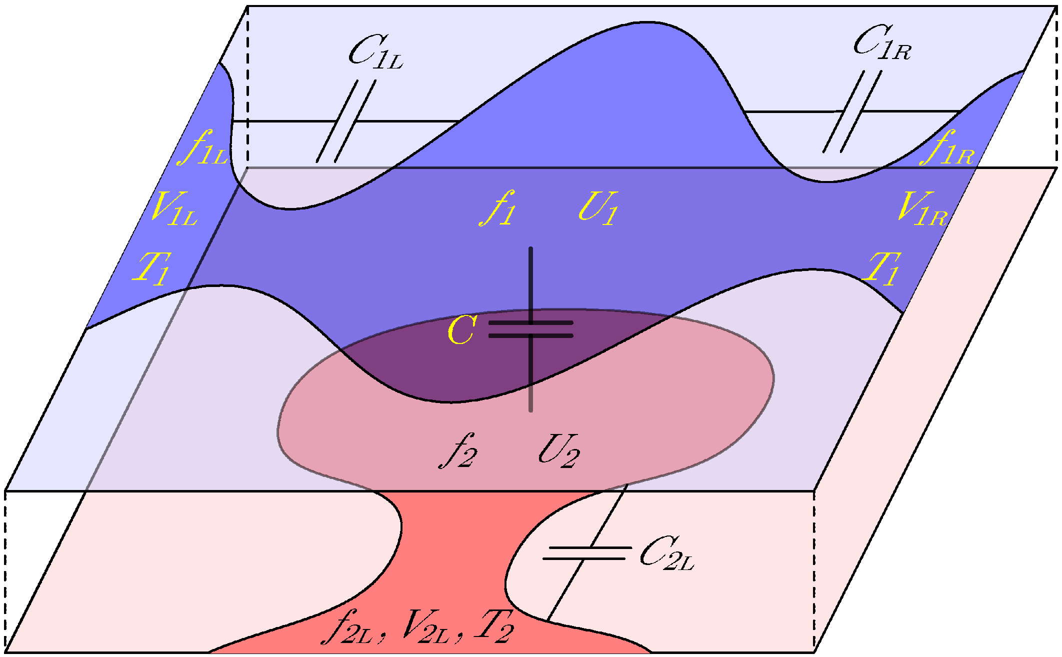
$$P_{\text{max}} = \frac{1}{4C_{\text{eff}}^2} \frac{G_r G_{\bar{r}}}{G_\Sigma} \left(\frac{G'_r}{G_r} - \frac{G'_{\bar{r}}}{G_{\bar{r}}} \right)^2 (k_{\text{B}}(T - T_R))^2$$

- Efficiency at maximum power

$$\eta_{\text{max}} = \frac{P_{\text{max}}}{\langle J_E \rangle} = \frac{1}{4C_{\text{eff}}} \frac{G_r G_{\bar{r}}}{G_\Sigma^2} \left(\frac{G'_r}{G_r} - \frac{G'_{\bar{r}}}{G_{\bar{r}}} \right)^2 k_{\text{B}} (T - T_R)$$

- Maximal power scales as $1/N$, efficiency scales as $1/N^2$

Double cavity



- Two capacitively coupled cavities
- Hot gate reservoir
- Investigate heat to current conversion

Double cavity: Current

Symmetric setup $C_{1\Sigma} = C_{2\Sigma} = C_\Sigma$, $\nu_{1\mathrm{F}} = \nu_{2\mathrm{F}} = \nu_\mathrm{F}$
Limit $G'_r \ll G_r$

$$\langle I_r \rangle = \frac{1}{C_\mathrm{eff}} \frac{G_{1\Sigma} G_{2\Sigma}}{G_{1\Sigma} + G_{2\Sigma}} \frac{G_{1r} G'_{1\bar{r}} - G_{1\bar{r}} G'_{1r}}{(G_{1\Sigma})^2} k_\mathrm{B} (T_1 - T_2)$$

- Effective capacitance describes coupling of cavities
- Second term: Maximal for cavities with identical transmissions
- Third term: Asymmetries
- Current linear in temperature difference

Double cavity: Efficiency

- Heat current between cavities

$$\langle J_E \rangle = \frac{1}{C_{\text{eff}}} \frac{G_{1\Sigma} G_{2\Sigma}}{G_{1\Sigma} + G_{2\Sigma}} k_{\text{B}} (T_i - T_{\bar{i}})$$

- Maximal extracted power

$$P_{\text{max}} = \frac{1}{4C_{\text{eff}}^2} \left(\frac{G_{1\Sigma} G_{2\Sigma}}{G_{1\Sigma} + G_{2\Sigma}} \right)^2 \frac{G_{1\Sigma}}{G_{1r} G_{1\bar{r}}} (\Lambda_{ir\bar{r}})^2 (k_{\text{B}} (T_1 - T_2))^2$$

- Efficiency at maximum power

$$\eta_{\text{max}} = \frac{P_{\text{max}}}{\langle J_E \rangle} = \frac{1}{4C_{\text{eff}}} \frac{G_{1\Sigma} G_{2\Sigma}}{G_{1\Sigma} + G_{2\Sigma}} \frac{G_{1\Sigma}}{G_{1r} G_{1\bar{r}}} (\Lambda_{ir\bar{r}})^2 k_{\text{B}} (T_1 - T_2)$$

- Maximal power scales as $1/N$, efficiency scales as $1/N^2$

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Summary

Summary & Conclusions

- I-V characteristics of a single cavity
- Heat-driven current by hot resistor or in a double cavity
- Heat to charge conversion needs symmetry breaking
- Maximal efficiency at maximum power, but $P \propto 1/N$ and $\eta \propto 1/N^2$

- Parallel setup of single-channel systems preferrable over many-channel system
- Energy-dependence of transmissions crucial for large efficiency