

Buckled beam oscillators for vibration harvesting

*Nonlinear Technologies for
Efficient Vibration Energy Harvesting*

Francesco Cottone

Marie Curie Research Fellow
(FP7-PEOPLE-2010 IEF – NEHSTech project)

ESIEE Université de Paris-Est &
NiPS Lab group, University of Perugia

ZEROPOWER WORKSHOP
26th -27th October 2011 Cork, Ireland

Outline

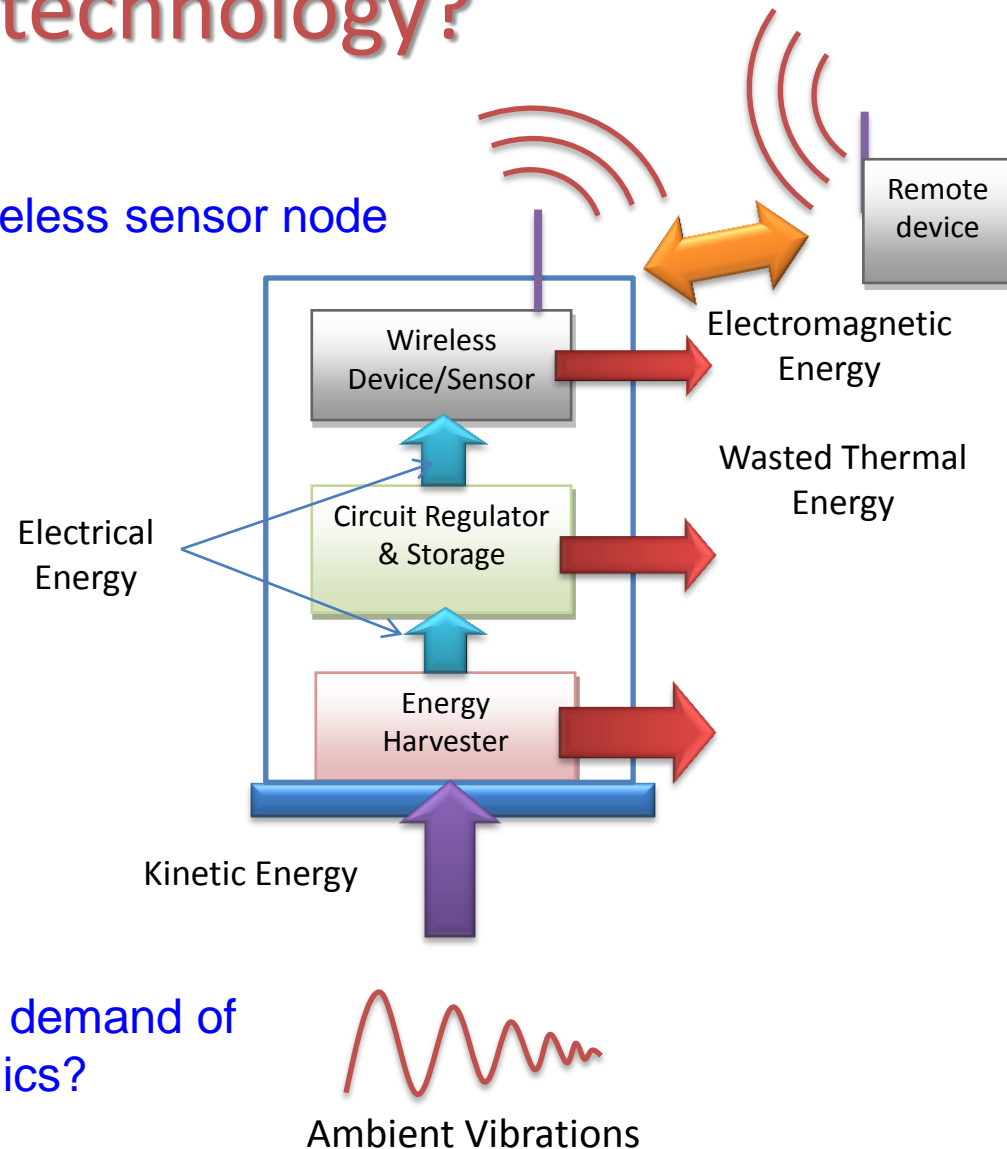
- Vibration energy harvesting and power requirements
- Bistable nonlinear oscillators vs resonant systems
- A buckled piezoelectric beam
- Theoretical model and experimental results
- Conclusions and future work

Vibration energy harvesting: a realistic technology?

Desirable power budget of a smart wireless sensor node

- Wireless node and sensors
 - RF antenna = $20\mu W$
 - Microprocessor = $20\mu W$
 - DSP = $20\mu W$
 - Sensor = $20\mu W$
- Circuit regulator = $20\mu W$
- Wasted heat ? = $20-100\mu W$

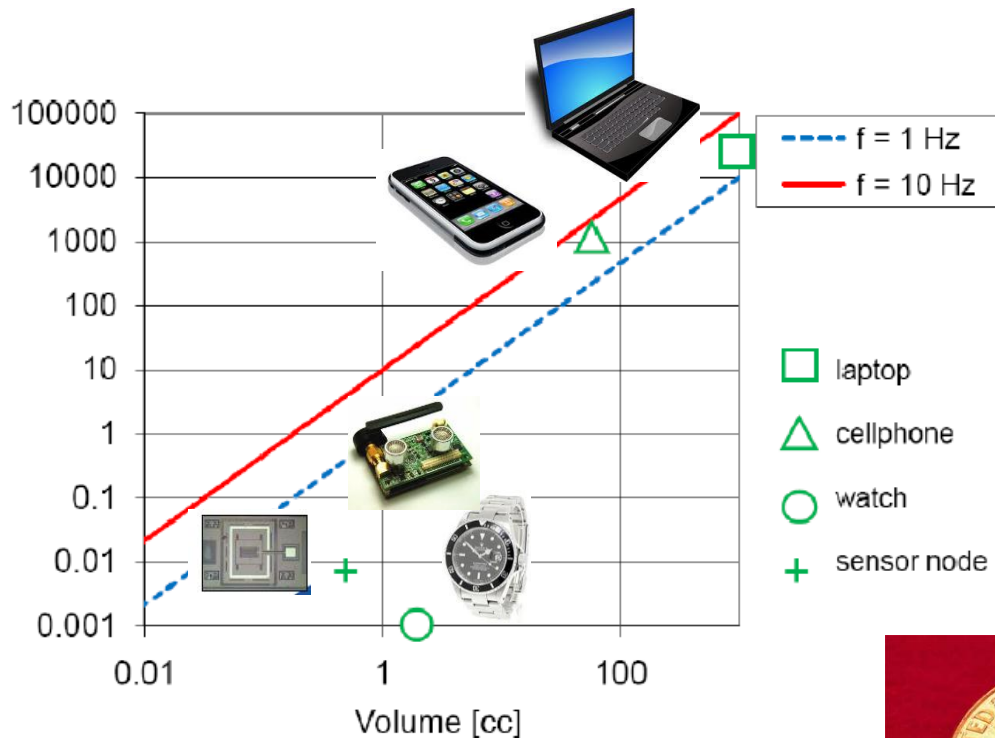
Energy Harvesting Generator
must provide at least $100-300\mu W$ per cm^3



Do vibration harvesters meet the power demand of
off-the-shelf commercial electronics?

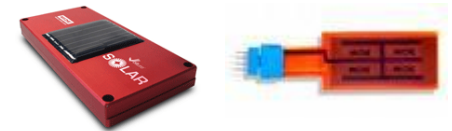
Vibration energy harvesting vs power requirements

Do next vibration harvesters meet the power demand of commercial electronics?



Perpetuum PMG17 (UK)
 Up to 45mW @ 1g rms (15Hz)
 GSM transmission capability

Mide' Voltare (USA)
 5mW @ 1grms (50Hz)



200 microwatts at 1.5g vibration @ 150Hz

University of Michigan (USA)
 2011



Mitcheson et al. (2008), *Proceedings of the IEEE* **96**(9): 1457-1486.

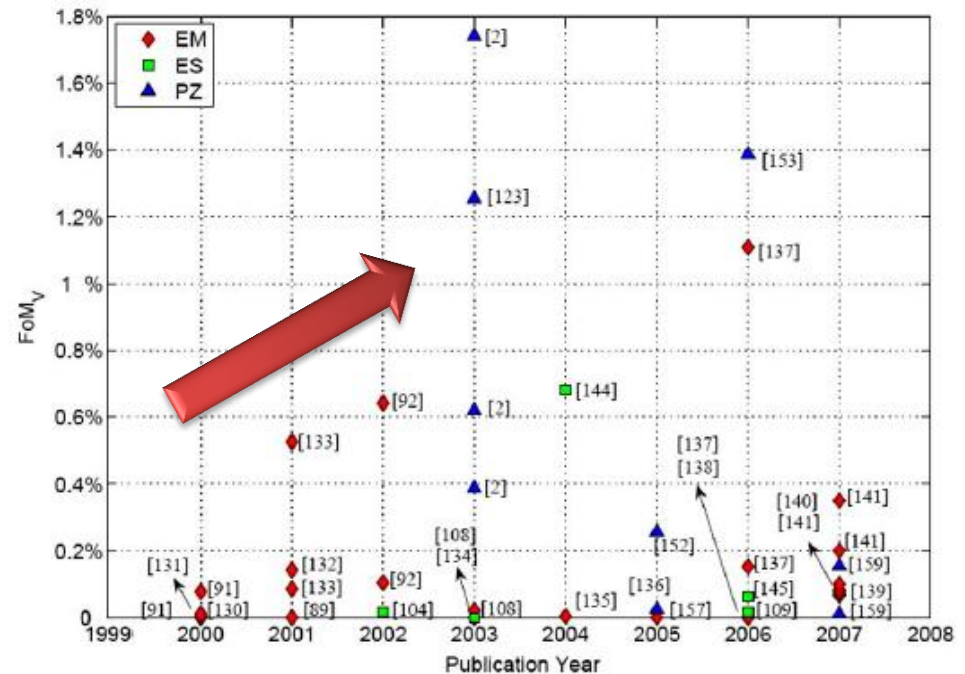
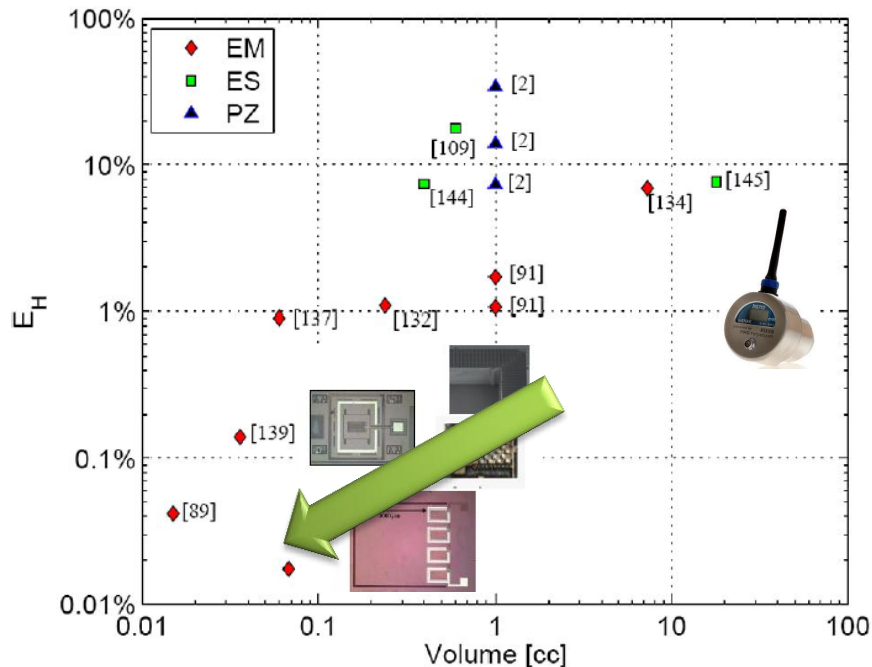
Available power from other sources

Energy Source	Harvested Power
Vibration/Motion	
Human	4 $\mu\text{W}/\text{cm}^2$
Industry	100 $\mu\text{W}/\text{cm}^2$
Temperature Difference	
Human	25 $\mu\text{W}/\text{cm}^2$
Industry	1–10 mW/cm^2
Light	
Indoor	10 $\mu\text{W}/\text{cm}^2$
Outdoor	10 mW/cm^2
RF	
GSM	0.1 $\mu\text{W}/\text{cm}^2$
WiFi	0.001 mW/cm^2

Texas Instruments, Energy Harvesting – White paper 2009

Size and performance of VEHs

- VEHs do not scale proportionally with dimensions!
- Definition of efficiency is still not complete, i.e. frequency bandwidth must be included.



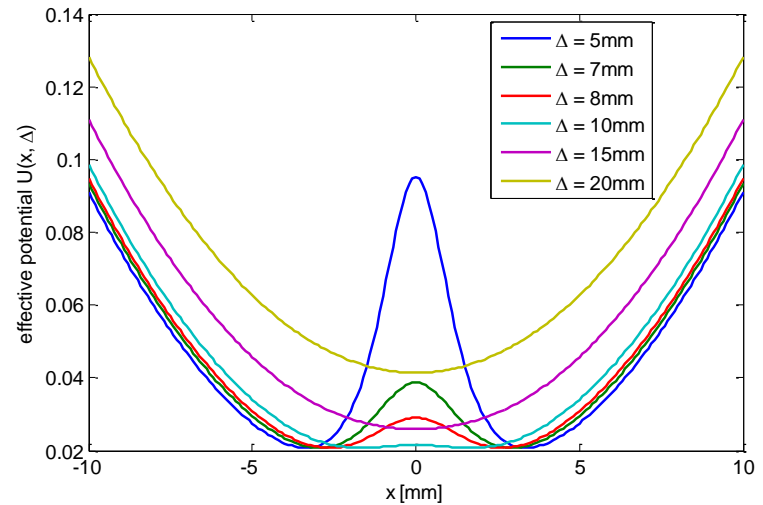
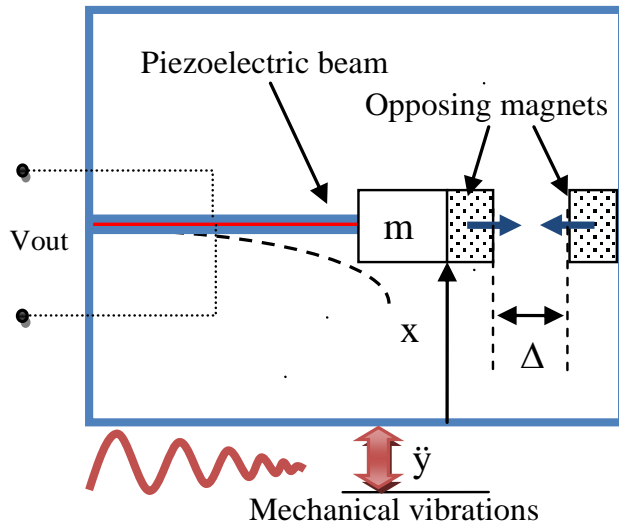
Mitcheson et al. (2008), *Proceedings of the IEEE* **96**(9): 1457-1486.

$$E_H = \frac{\text{Useful Power Output}}{\text{Maximum Possible Output}} = \frac{\text{Useful Power Output}}{\frac{1}{2} Y_0 Z_l \omega^3 m}$$

$$FoM_V = \frac{\text{Useful Power Output}}{\frac{1}{16} Y_0 \rho_{Au} V_0 l^4 \omega^3}$$

Bistable oscillators vs resonant systems for vibration energy harvesting

Real vibrations are variable in time and frequency



Repulsive force

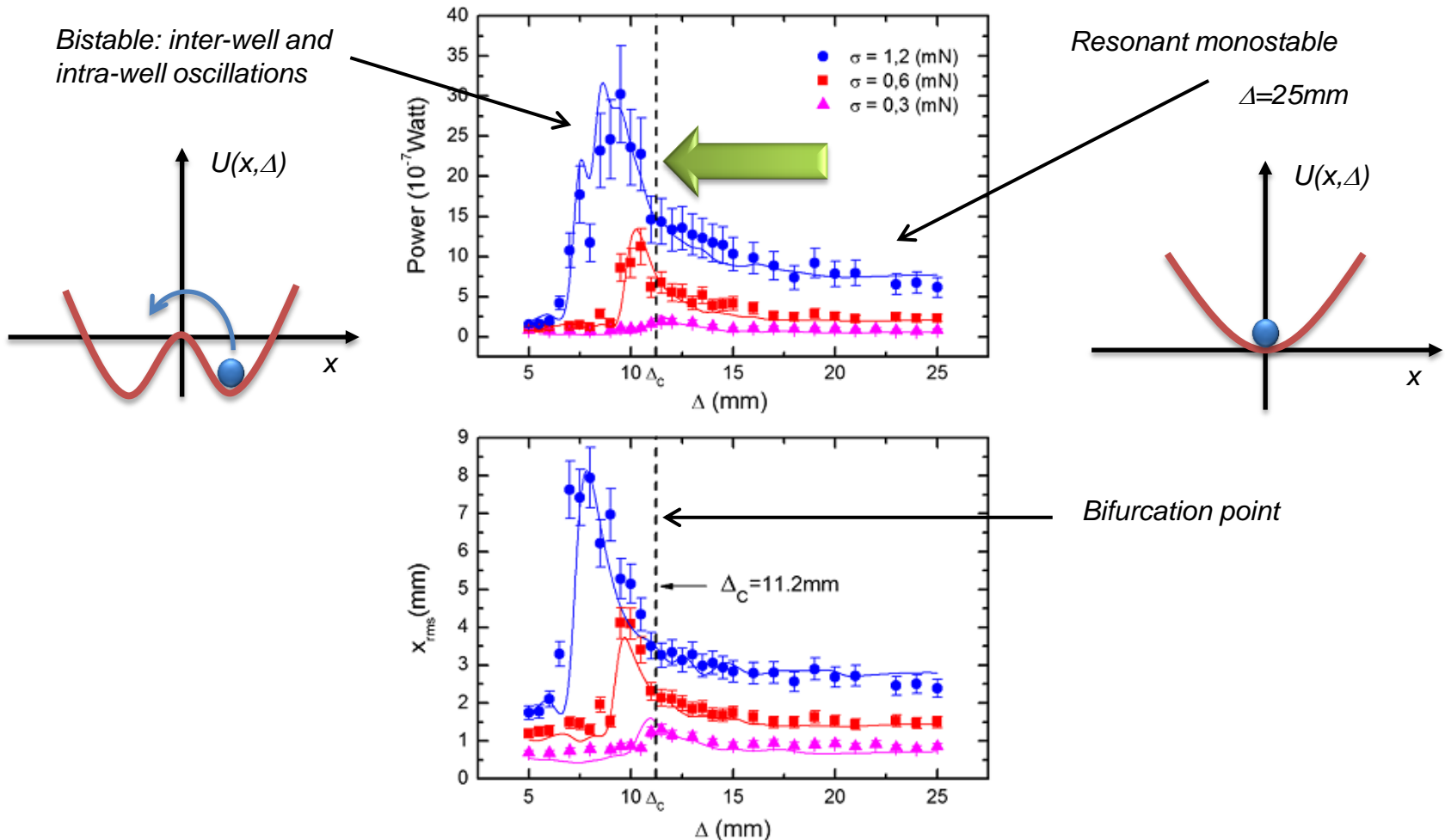
$$F_m = \frac{3\mu_0 M_1 M_2^2}{2\pi} \frac{x}{(x^2 + \Delta^2)^{5/2}}$$

$$U(x, \Delta) = \frac{1}{2} K_{eff} (x^2 + \Delta^2) + \frac{\mu_0}{2\pi} \frac{M_1 M_2}{(x^2 + \Delta^2)^{3/2}}$$

Governing equations of single-DOF model
(mass normalized)

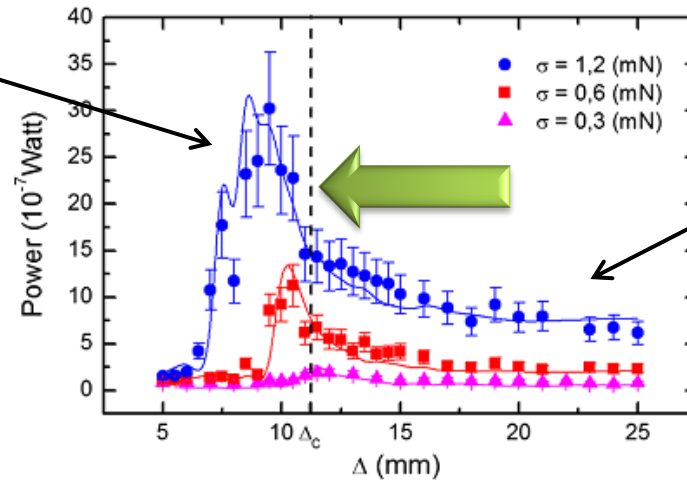
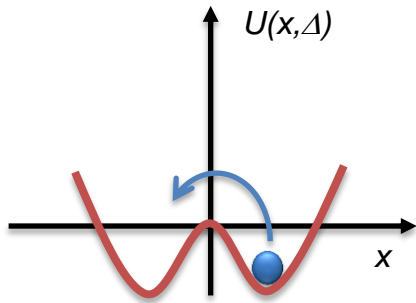
$$\begin{cases} \ddot{x}(t) = -K_{eff} x(t) - 2\delta\omega_0 \dot{x}(t) + \frac{3\mu_0 M_1 M_2}{2\pi} \frac{x(t)}{(x(t)^2 + \Delta^2)^{5/2}} - K_v V(t) - \sigma \xi(t) \\ \dot{V}(t) = K_c \dot{x}(t) - \frac{V(t)}{\tau}; \quad \tau = 1 / R_L C_p \end{cases}$$

Bistable oscillators vs resonant systems for vibration energy harvesting



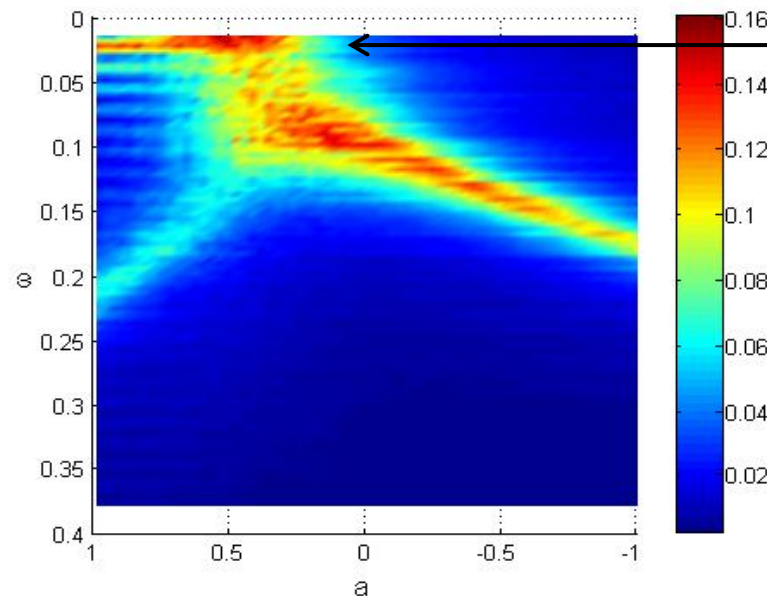
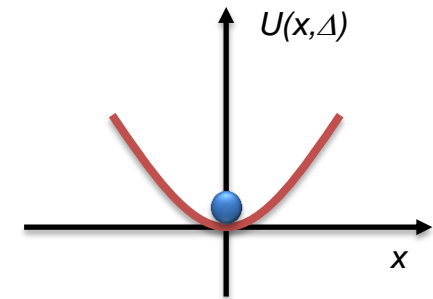
Bistable oscillators vs resonant systems for vibration energy harvesting

Bistable: inter-well and intra-well oscillations



Resonant monostable

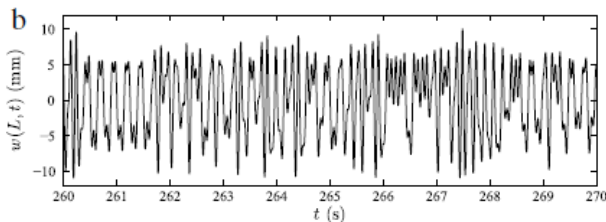
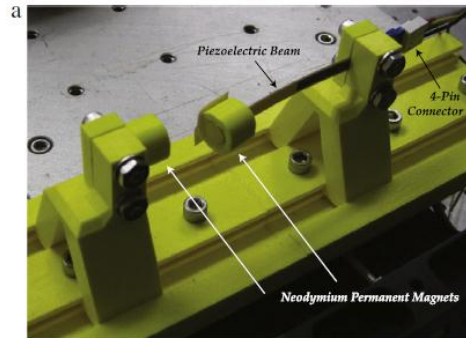
$\Delta = 25 \text{ mm}$



Bandwidth enhancement

Bistable oscillators vs resonant systems for vibration energy harvesting

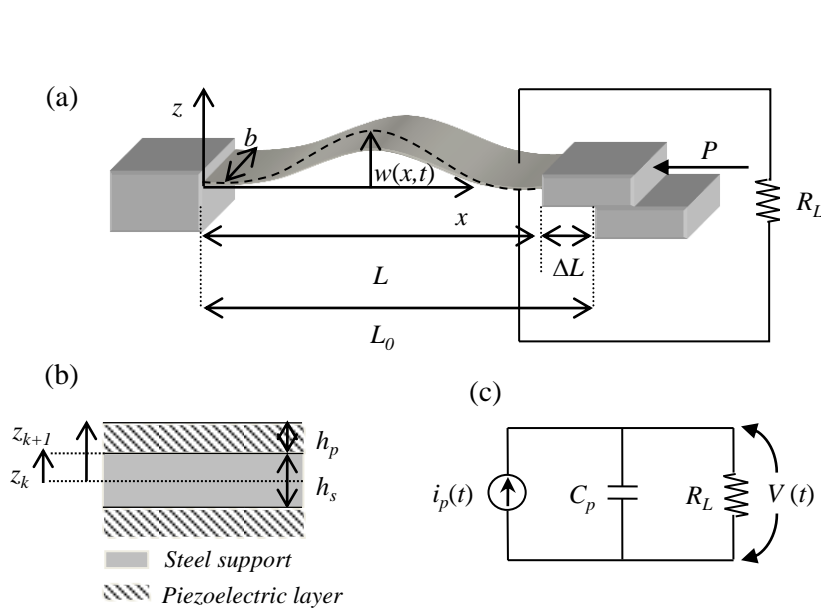
Independent research groups validated the superiority of nonlinear bistable oscillators for vibrations energy harvesting



- Erturk, A. and D. Inman (2010). "Broadband piezoelectric power generation on high-energy orbits of the bistable Duffing oscillator with electromechanical coupling." Journal of Sound and Vibration.
- Arrieta, A., P. Hagedorn, et al. (2010). "A piezoelectric bistable plate for nonlinear broadband energy harvesting." Applied Physics Letters 97: 104102
- Barton, Burrow, et al. (2010). "Energy harvesting from vibrations with a nonlinear oscillator." Journal of vibration and acoustics 132: 021009..
- Mann, B. and B. Owens (2010). "Investigations of a nonlinear energy harvester with a bistable potential well." Journal of Sound and Vibration 329(9): 1215-1226.
- Stanton et al. (2010). "Nonlinear dynamics for broadband energy harvesting: Investigation of a bistable piezoelectric inertial generator." Physica D: Nonlinear Phenomena 239(10): 640-653.

Bistable oscillators vs resonant systems for vibration energy harvesting

Buckled piezoelectric beams



$$S_x = \varepsilon_{xx}^0 + z \varepsilon_{xx}^1, \quad \text{with} \quad \varepsilon_{xx}^0 = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2, \quad \varepsilon_{xx}^1 = -\frac{\partial^2 w}{\partial x^2},$$

Von Kàrman nonlinear relations

associated to mid-plane displacement field (u, w) respectively along x and z -axis

$$w(x, t) = w_1(x) + v(x, t) \quad v(x, t) = \sum_{i=1}^N r_i(t) \varphi_i(x)$$

$$K = \frac{1}{2} \rho_s A_s \int_0^L \left(\frac{\partial w}{\partial t} + \frac{dz}{dt} \right)^2 dx + \frac{n}{2} \rho_p A_p \int_0^{L_p} \left(\frac{\partial w}{\partial t} + \frac{dz}{dt} \right)^2 dx$$

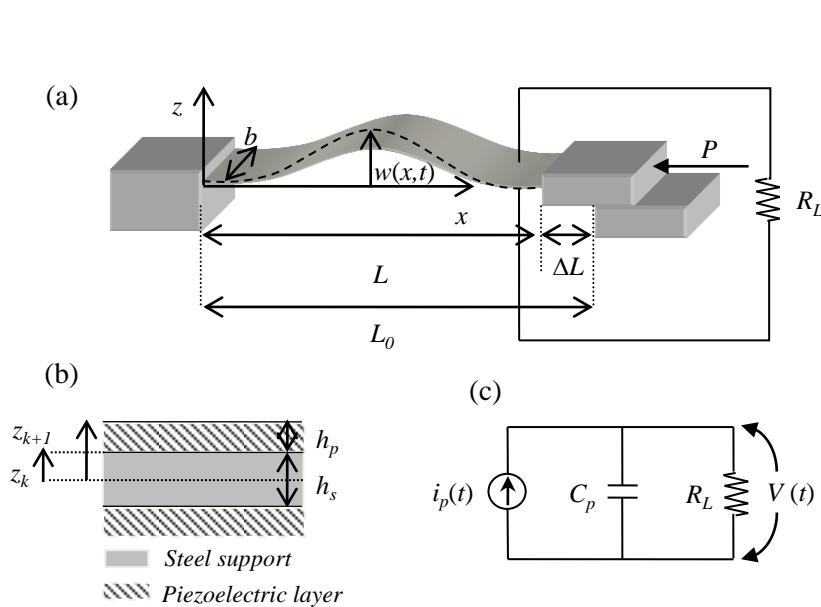
Kinetic energy

$$\Pi = \frac{1}{2} \int_0^L A (\varepsilon_{xx}^0)^2 dx + \int_0^L B \varepsilon_{xx}^0 \varepsilon_{xx}^1 dx + \frac{1}{2} \int_0^L D (\varepsilon_{xx}^1)^2 dx - \int_0^L N_p \varepsilon_{xx}^0 dx - \int_0^L M_p \varepsilon_{xx}^1 dx - b h_p \int_0^L \varepsilon_{zz}^S E_z^2 dx - W.$$

Strain energy

Bistable oscillators vs resonant systems for vibration energy harvesting

Buckled piezoelectric beams



$$S_x = \varepsilon_{xx}^0 + z\varepsilon_{xx}^1, \quad \text{with} \quad \varepsilon_{xx}^0 = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2, \quad \varepsilon_{xx}^1 = -\frac{\partial^2 w}{\partial x^2},$$

Von Kàrman nonlinear relations

associated to mid-plane displacement field (u, w) respectively along x and z -axis

$$w(x, t) = w_1(x) + v(x, t) \quad v(x, t) = \sum_{i=1}^N r_i(t) \varphi_i(x)$$

$$w(x, t) = h_0 \varphi_1(x) + r_1(t) \varphi_1(x) \quad \text{the first vibration mode}$$

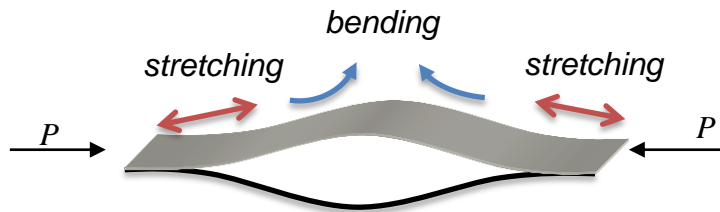
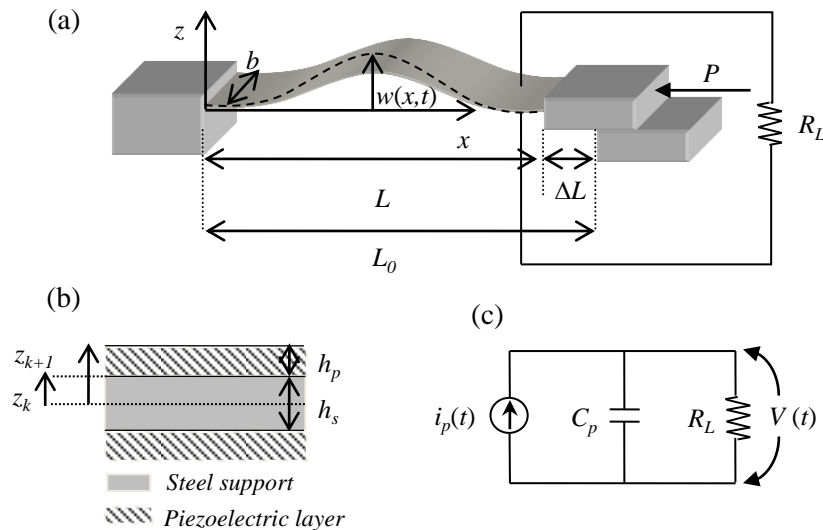
$$\mathcal{L}(q, \dot{q}, \dot{\lambda}) = \frac{1}{2} m (\dot{q}^2 + \dot{z}^2) + \eta \dot{q} \dot{z} - \frac{1}{4} k_3 q^4 - \frac{1}{2} (k_2 + k_1 \dot{\lambda}) q^2 + k_0 \dot{\lambda} q + \frac{1}{4} C_p \dot{\lambda}^2 - P d_b,$$

The Lagrangian of single DOF model for the midpoint

vibration energy harvesting

Buckled piezoelectric beams

$$w(x, t) = w_1(x) + v(x, t)$$



the initial buckling shape function is

$$\psi(x) = h_0 (1 - \cos(2\pi x / L)) / 2$$

by applying Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = F(t), \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\lambda}}\right) - \frac{\partial \mathcal{L}}{\partial \lambda} = I(t)$$

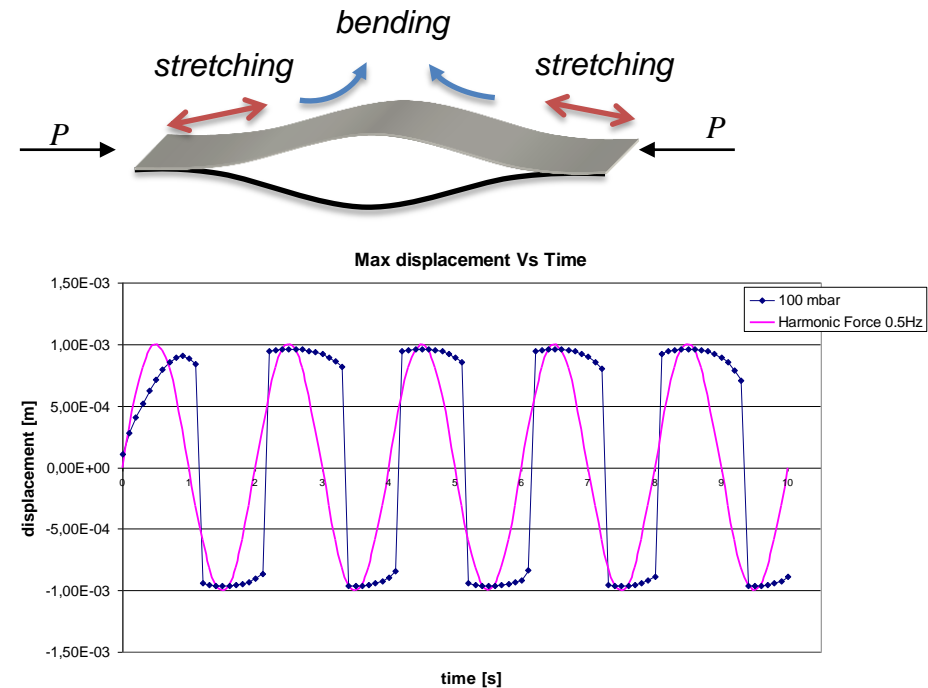
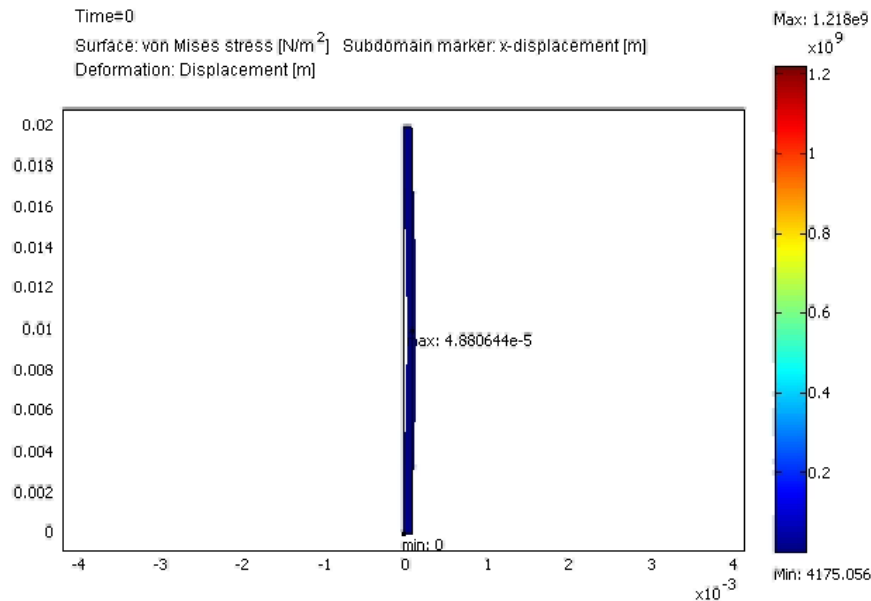
gives two coupled second order nonlinear differential equations governing the motion of the midpoint linked to output voltage across the resistive load $V(t)=-\dot{\lambda}(t)$

$$\begin{cases} m\ddot{q} + c\dot{q} + k_3q^3 + (k_2 + k_1\dot{\lambda})q - k_0\dot{\lambda} = -\eta\ddot{z}, \\ \frac{1}{2}C_p\ddot{\lambda} + \frac{\dot{\lambda}}{R_L} = k_1q\dot{q} - k_0\dot{q}. \end{cases}$$

Bistable oscillators vs resonant systems for vibration energy harvesting

Buckled piezoelectric beams

FEA time-dependent simulations with sinusoidal excitation

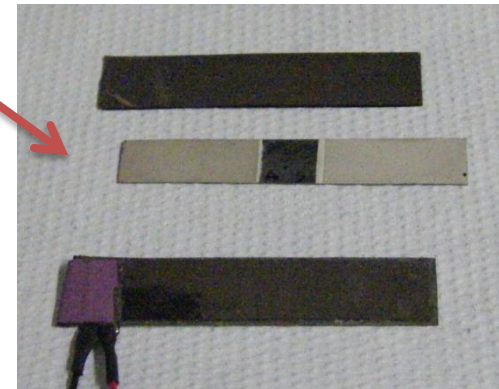
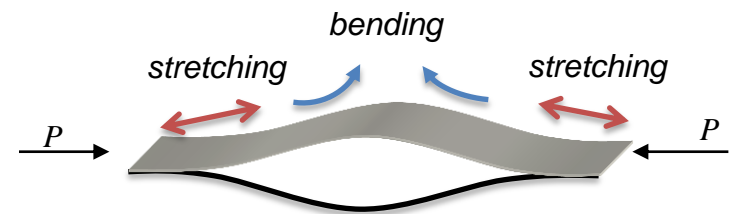
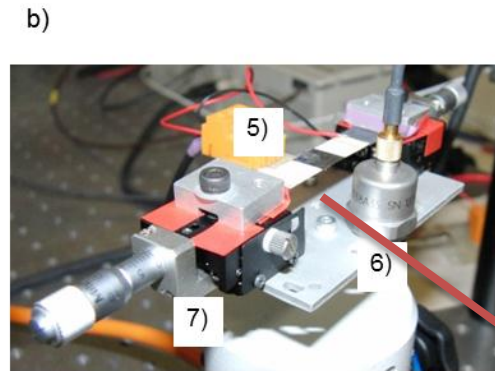
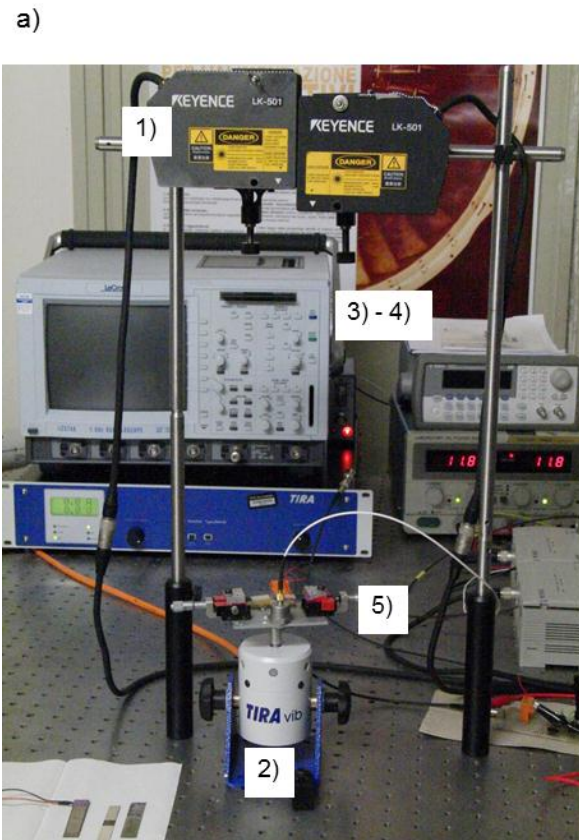


contracting the clamping of 0.1mm we induce snap-through bistable response

- F Cottone, PhD Thesis, *Nonlinear Piezoelectric Generators for Vibration Energy Harvesting*

Bistable oscillators vs resonant systems for vibration energy harvesting

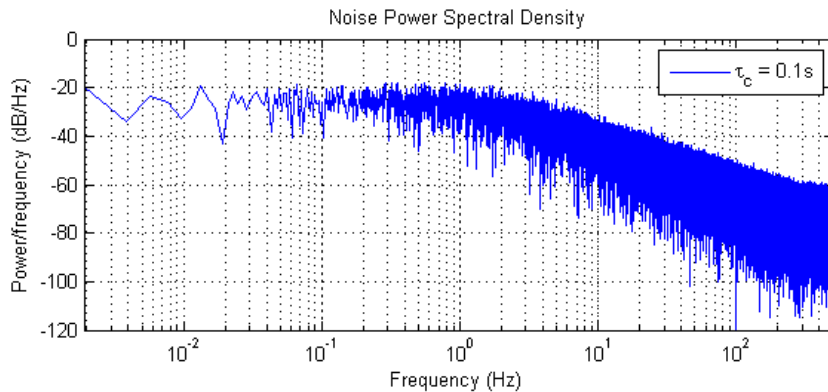
Experimental apparatus (NiPS Lab. Perugia)



- 1) Laser displacement sensors
- 2) Shaker
- 3) Spectrum analyzer
- 4) Acquisition systems, signal generators and power amplifier
- 5) Piezoelectric beam
- 6) Accelerometer
- 7) Micrometric stage

Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental test

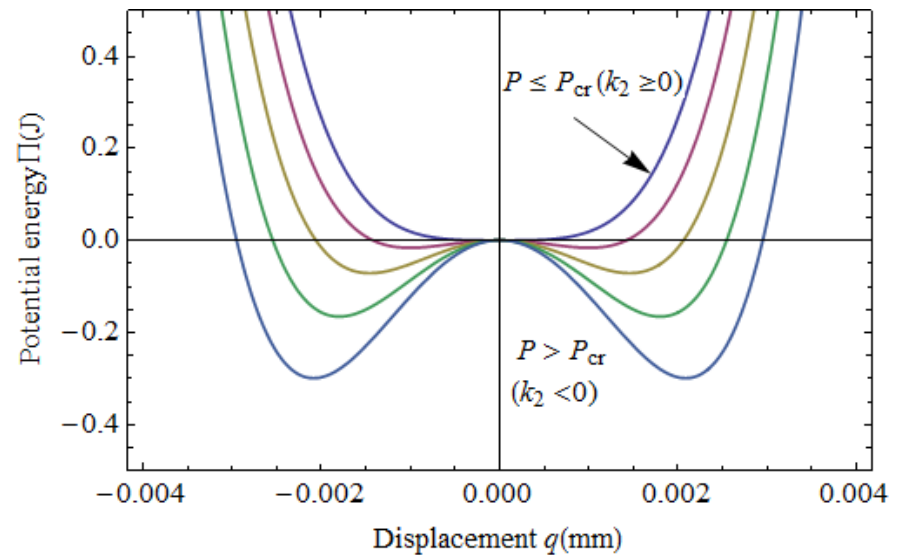
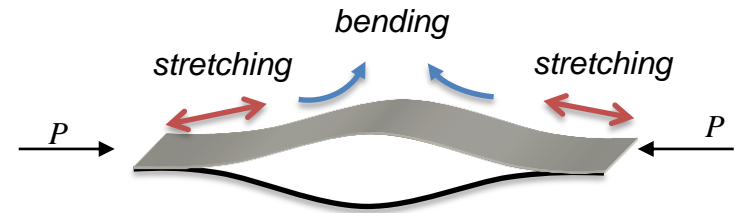


Exp. Corr. Gaussian Noise

$$\langle \xi_e(t) \xi_e(t') \rangle = \langle \xi_e^2 \rangle \exp[-|t - t'| / \tau_c] \quad \tau = 0.1 - 0.001 \text{ s}$$

*In the postbuckled static configuration
the elastic energy results to be*

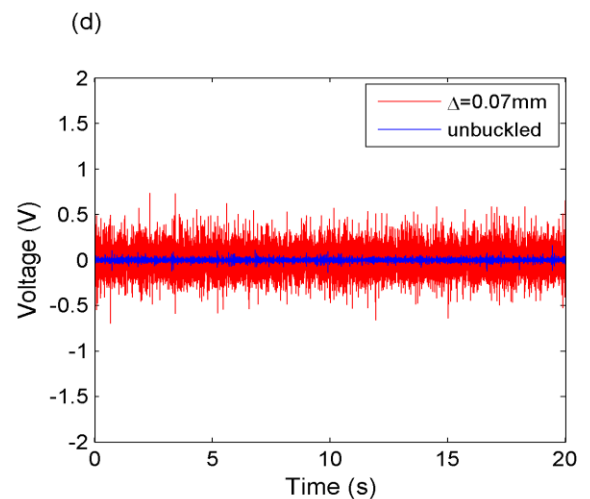
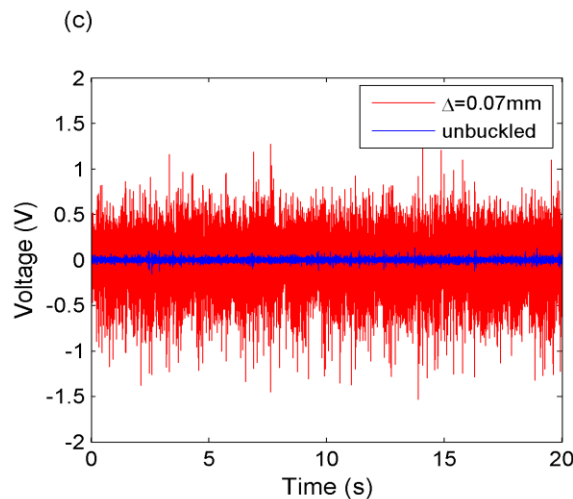
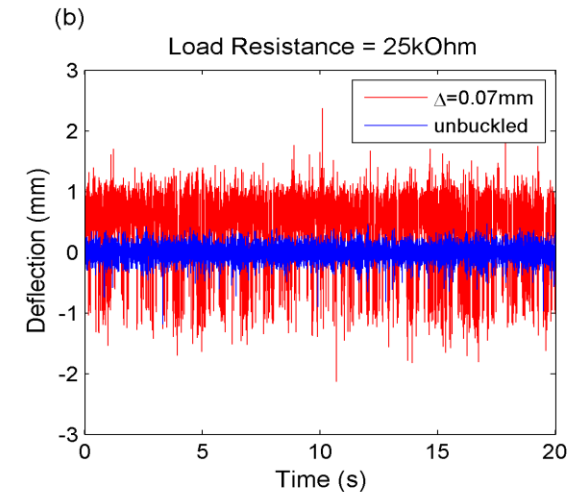
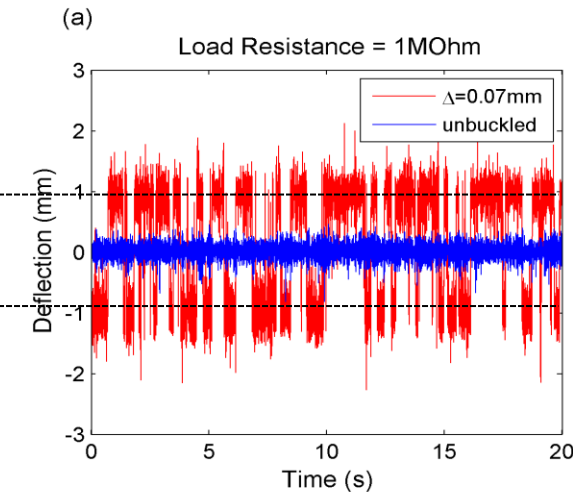
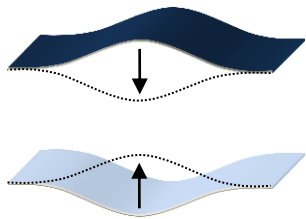
$$\Pi(q) = \frac{1}{4} k_3 q^4 + \frac{1}{2} k_2 q^2 + P_{cr} d_b.$$



Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental and numerical results

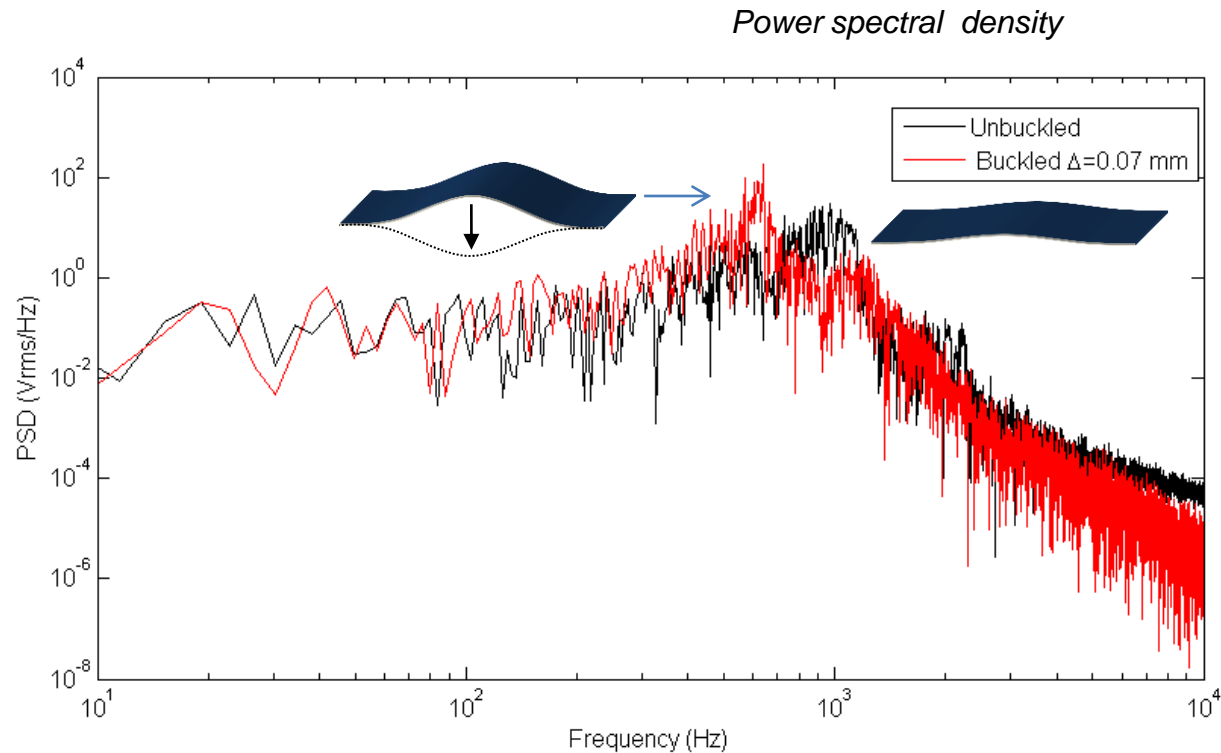
Snapping between buckled states



Bistable oscillators vs resonant systems for vibration energy harvesting

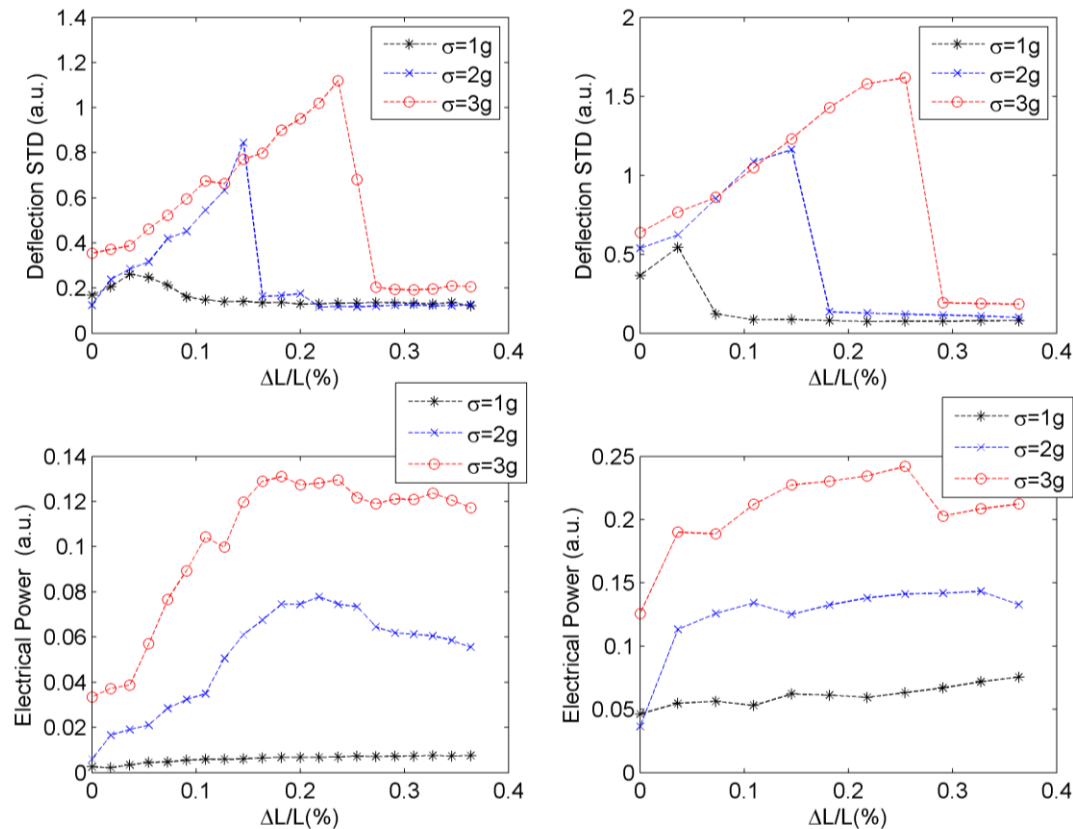
Experimental and numerical results

Snapping between buckled states



Bistable oscillators vs resonant systems for vibration energy harvesting

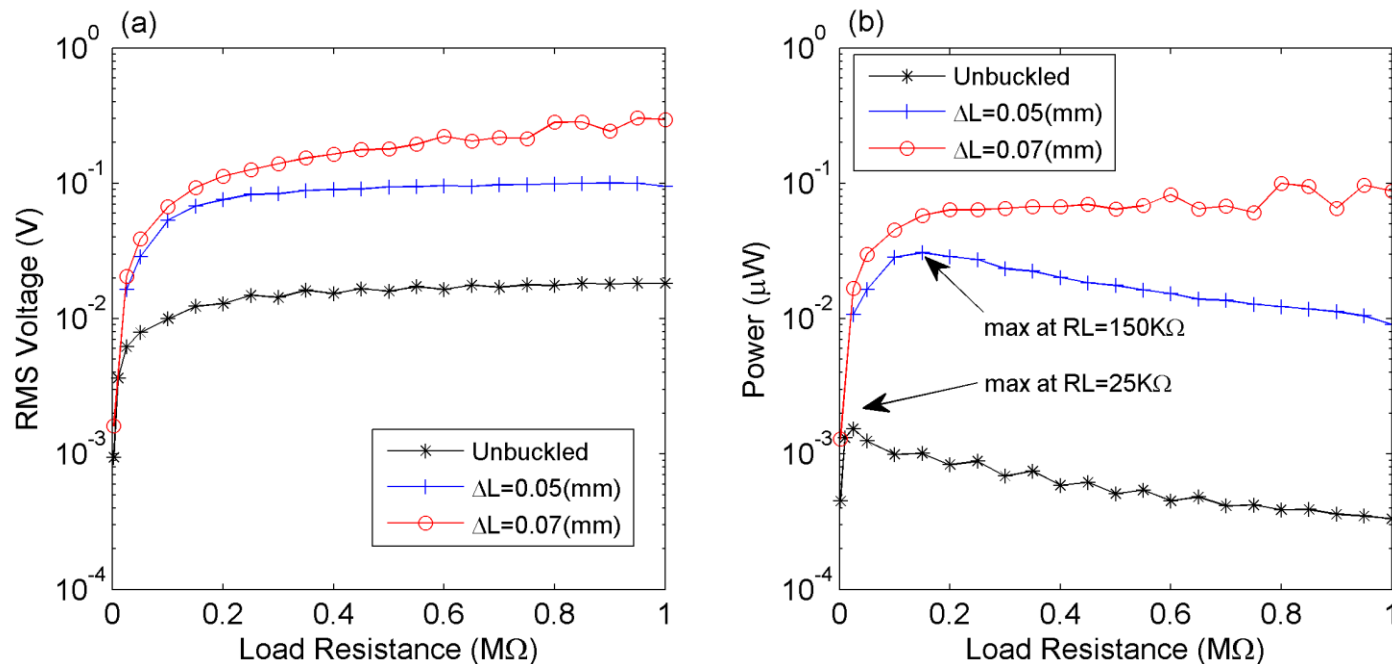
Experimental and numerical results



Submitted for publication to *Smart Materials & Structures* journal:
"Piezoelectric buckled beams for random vibrations energy harvesting"
F Cottone, L Gammaitoni, H Vocca, M Ferrari, V Ferrari

Bistable oscillators vs resonant systems for vibration energy harvesting

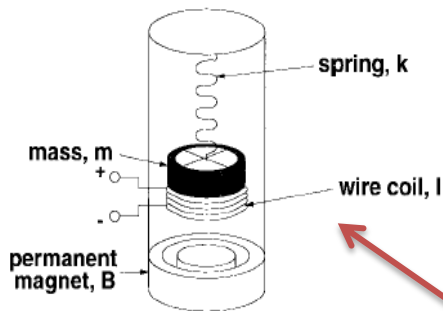
Experimental and numerical results



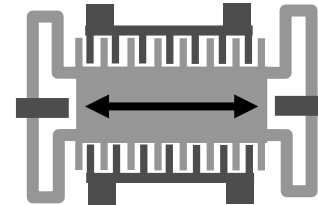
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Buckled beam concept for other transduction methods

Electromagnetic

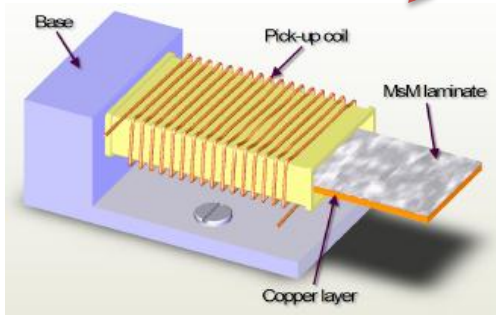


Electrostatic/Capacitive

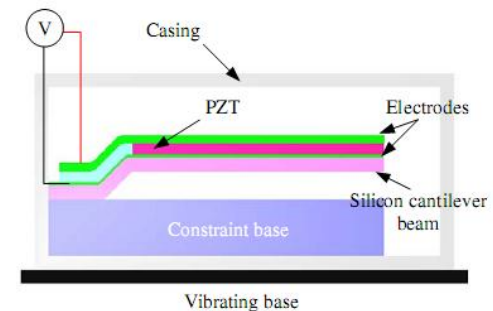


Vibration Harvesting Generator

Magnetostrictive



Piezoelectric

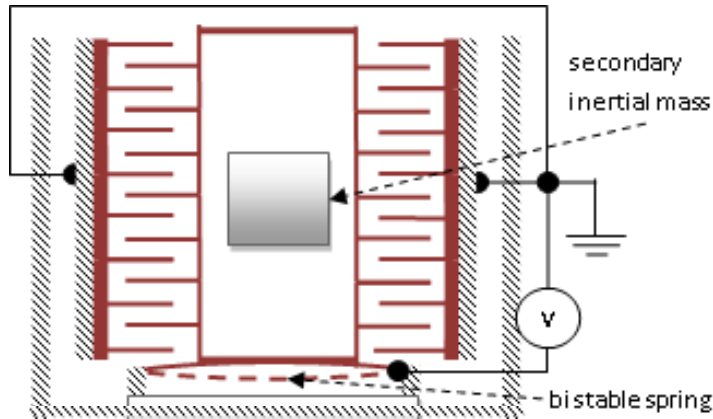


Ferroelectric materials: PZT, PVDF, AIN

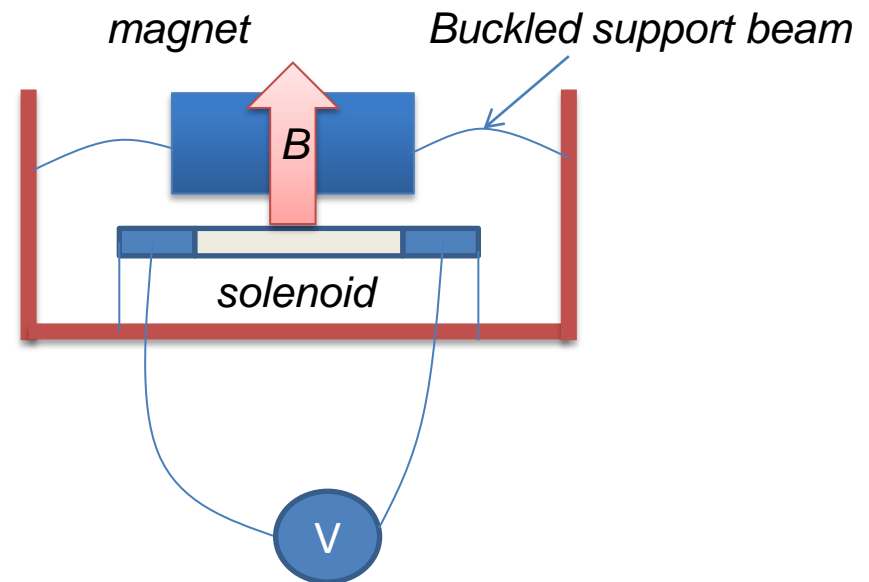
Ferromagnetic materials: crystalline alloy Terfenol-D
amorphous metallic glass Metglas ($\text{Fe}_8\text{B}_{13.5}\text{Si}_{3.5}\text{C}_2$).

Buckled beam concept for other transduction methods

Silicon-based MEMS comb-drive capacitor



Electromagnetic buckled beam system



NEHSTech IEF Marie Curie Project at ESIEE
University of Paris Est - prof. P. Basset and F. Cottone

Conclusions

- Bistable harvesters have been confirmed to outperform resonant systems under random and harmonic vibration source both in terms of frequency response and voltage amplitude.
- A buckled piezoelectric beam has been theoretically modeled, also considering in-plane stretching effects, and experimentally investigated.
 - Numerical results predict in good qualitative agreement the physical behaviour.
- The nonlinear buckled configuration has been demonstrated to be more efficient in terms of power density and frequency bandwidth. The overall electrical power results multiplied by more than a factor 3x.
- A counterintuitive not decreasing voltage occurs even when the systems oscillates within one of the two minima at high compression load.

Future work

- Validation of the buckled beam concept is envisaged for other type of conversion methods: electrostatic, electrodynamic and/or magnetostrictive.
- Scaling down to millimetric- and/or micro-scale must be further investigated.
- Vibration-driven microgenerator must be tested in real scenarios to assess the efficacy of the proposed concept. For example by using real vibration database.
- Integrated wireless sensor node system with power aware electronics and nonlinear vibrational generators are expected to be developed and validated.

Thanks for your attention!



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NiPS Laboratory
Noise in Physical Systems