Buckled beam oscillators for vibration harvesting

Nonlinear Technologies for Efficient Vibration Energy Harvesting

Francesco Cottone

Marie Curie Research Fellow
(FP7-PEOPLE-2010 IEF – NEHSTech project)

ESIEE Université de Paris-Est &
NiPS Lab group, University of Perugia

ZEROPOWER WORKSHOP
26th -27th October 2011 Cork, Ireland
Outline

• Vibration energy harvesting and power requirements
• Bistable nonlinear oscillators vs resonant systems
• A buckled piezoelectric beam
• Theoretical model and experimental results
• Conclusions and future work
Vibration energy harvesting: a realistic technology?

Desirable power budget of a smart wireless sensor node

- Wireless node and sensors
  - RF antenna = 20μW
  - Microprocessor = 20μW
  - DSP = 20μW
  - Sensor = 20μW

- Circuit regulator = 20μW
- Wasted heat ? = 20-100μW

Energy Harvesting Generator must provide at least 100-300μW per cm³

Do vibration harvesters meet the power demand of off-the-shelf commercial electronics?
Vibration energy harvesting vs power requirements

Do next vibration harvesters meet the power demand of commercial electronics?

Perpetuum PMG17 (UK)
Up to 45mW @ 1g rms (15Hz)
GSM transmission capability

Mide’ Volture (USA)
5mW @ 1grms (50Hz)

200 microwatts at 1.5g vibration @ 150Hz
University of Michigan (USA) 2011

Mitcheson et al. (2008), Proceedings of the IEEE 96(9): 1457-1486.
Available power from other sources

<table>
<thead>
<tr>
<th>Energy Source</th>
<th>Harvested Power</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vibration/Motion</strong></td>
<td></td>
</tr>
<tr>
<td>Human</td>
<td>4 μW/cm²</td>
</tr>
<tr>
<td>Industry</td>
<td>100 μW/cm²</td>
</tr>
<tr>
<td><strong>Temperature Difference</strong></td>
<td></td>
</tr>
<tr>
<td>Human</td>
<td>25 μW/cm²</td>
</tr>
<tr>
<td>Industry</td>
<td>1–10 mW/cm²</td>
</tr>
<tr>
<td><strong>Light</strong></td>
<td></td>
</tr>
<tr>
<td>Indoor</td>
<td>10 μW/cm²</td>
</tr>
<tr>
<td>Outdoor</td>
<td>10 mW/cm²</td>
</tr>
<tr>
<td><strong>RF</strong></td>
<td></td>
</tr>
<tr>
<td>GSM</td>
<td>0.1 μW/cm²</td>
</tr>
<tr>
<td>WIFI</td>
<td>0.001 mW/cm²</td>
</tr>
</tbody>
</table>

*Texas Instruments, Energy Harvesting – White paper 2009*
Size and performance of VEHs

- VEHs do not scale proportionally with dimensions!
- Definition of efficiency is still not complete, i.e. frequency bandwidth must be included.

\[ E_H = \frac{\text{Useful Power Output}}{\text{Maximum Possible Output}} = \frac{1}{2} Y_0 Z_0 \omega^3 m \]

\[ \text{FoM}_V = \frac{\text{Useful Power Output}}{\frac{1}{16} Y_0 \rho_A V_0 l^3 \omega^3} \]

Mitcheson et al. (2008), Proceedings of the IEEE 96(9): 1457-1486.
**Bistable oscillators vs resonant systems for vibration energy harvesting**

Real vibrations are variable in time and frequency

![Diagram showing mechanical vibrations, repulsive force, and governing equations of single-DOF model.](image)

\[ F_m = \frac{3\mu_0 M_1 M_2}{2\pi} \frac{x}{(x^2 + \Delta^2)^{3/2}} \]

\[ U(x,\Delta) = \frac{1}{2} K_{eff} \left( x^2 + \Delta^2 \right) + \frac{\mu_0 M_1 M_2}{2\pi} \frac{M_1 M_2}{(x(t)^2 + \Delta^2)^{3/2}} \]

\[
\begin{align*}
\ddot{x}(t) &= -K_{eff} x(t) - 2\delta \omega_0 \dot{x}(t) + \frac{3\mu_0 M_1 M_2}{2\pi} \frac{x(t)}{(x(t)^2 + \Delta^2)^{5/2}} - K_v V(t) - \sigma \xi(t) \\
\dot{V}(t) &= K_c \dot{x}(t) - \frac{V(t)}{\tau}; \quad \tau = 1 / R_L C_p
\end{align*}
\]
Bistable oscillators vs resonant systems for vibration energy harvesting

Bistable: inter-well and intra-well oscillations

Resonant monostable

$\Delta = 25\text{mm}$

$U(x, \Delta)$

Bifurcation point

$\Delta_c = 11.2\text{mm}$

$U(x, \Delta)$

$\Delta_c = 25\text{mm}$

Bistable oscillators vs resonant systems for vibration energy harvesting

Bistable: inter-well and intra-well oscillations

Resonant monostable

\( \Delta = 25 \text{mm} \)

\( U(x, \Delta) \)

Bandwidth enhancement

\[ U(x, \Delta) = \cdots \]

\[ \Delta = \cdots \]

\[ \sigma = 1.2 \text{ (mN)} \]

\[ \sigma = 0.6 \text{ (mN)} \]

\[ \sigma = 0.3 \text{ (mN)} \]

Power (\(10^{-7}\) Watt)

\( \Delta \) (mm)
Bistable oscillators vs resonant systems for vibration energy harvesting

Independent research groups validated the superiority of nonlinear bistable oscillators for vibrations energy harvesting


Bistable oscillators vs resonant systems for vibration energy harvesting

Buckled piezoelectric beams

\[ S_x = \varepsilon_{xx}^0 + z\varepsilon_{xx}^1, \quad \text{with} \quad \varepsilon_{xx}^0 = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2, \quad \varepsilon_{xx}^1 = -\frac{\partial^2 w}{\partial x^2}, \]

Von Kármán nonlinear relations

associated to mid-plane displacement field \((u,w)\) respectively along \(x\) and \(z\)-axis

\[ w(x,t) = w_1(x) + v(x,t) \quad \quad v(x,t) = \sum_{i=1}^{N} r_i(t) \phi_i(x) \]

\[ K = \frac{1}{2} \rho A \int_0^L \left( \frac{\partial w}{\partial t} + \frac{dz}{dt} \right)^2 dx + \frac{n}{2} \rho_p A_p \int_0^L \left( \frac{\partial w}{\partial t} + \frac{dz}{dt} \right)^2 dx \]

Kinetic energy

\[ \Pi = \frac{1}{2} \int_0^L A(\varepsilon_{xx}^0)^2 dx + \frac{1}{2} \int_0^L B(\varepsilon_{xx}^0)^2 dx + \frac{1}{2} \int_0^L D(\varepsilon_{xx}^1)^2 dx - \int_0^L N_p \varepsilon_{xx}^0 dx - \int_0^L M_p \varepsilon_{xx}^1 dx - bh_p \int_0^L \varepsilon_{zz}^2 E^2 dx - W. \]

Strain energy
Bistable oscillators vs resonant systems for vibration energy harvesting

**Buckled piezoelectric beams**

\[
S_x = \varepsilon_{xx}^0 + z\varepsilon_{xx}^1, \quad \text{with} \quad \varepsilon_{xx}^0 = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2, \quad \varepsilon_{xx}^1 = -\frac{\partial^2 w}{\partial x^2},
\]

Von Kàrman nonlinear relations associated to mid-plane displacement field \((u,w)\) respectively along \(x\) and \(z\)-axis

\[
w(x,t) = w_1(x) + v(x,t) \quad \text{and} \quad v(x,t) = \sum_{i=1}^{N} r_i(t) \phi_i(x)
\]

\[
w(x,t) = h_0\phi_1(x) + r_1(t)\phi_1(x) \quad \text{the first vibration mode}
\]

\[
\mathcal{L}(q, \dot{q}, \dot{\lambda}) = \frac{1}{2} m (\dot{q}^2 + \dot{z}^2) + \eta \dot{q} \ddot{z} - \frac{1}{4} k_3 q^4 - \frac{1}{2} \left( k_2 + k_1 \dot{\lambda} \right) q^2 + k_0 \dot{\lambda} q + \frac{1}{4} C_p \dot{\lambda}^2 - P d_b,
\]

The Lagrangian of single DOF model for the midpoint
Bistable oscillators vs resonant systems for vibration energy harvesting

Buckled piezoelectric beams

\[ w(x,t) = w_1(x) + v(x,t) \]

the initial buckling shape function is
\[ \psi(x) = h_0(1 - \cos(2\pi x / L)) / 2 \]

by applying Euler-Lagrange equations
\[
\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}}{\partial q} = F(t), \quad\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} \right) - \frac{\partial \mathcal{L}}{\partial \lambda} = I(t)
\]

\[
\begin{cases}
m\ddot{q} + c\dot{q} + k_3q^3 + \left(k_2 + k_1\dot{\lambda}\right)q - k_0\dot{\lambda} = -\eta\ddot{z}, \\
\frac{1}{2}C_p\ddot{\lambda} + \frac{\dot{\lambda}}{R_L} = k_1q\dot{q} - k_0\dot{q}.
\end{cases}
\]

gives two coupled second order nonlinear differential equations governing the motion of the midpoint linked to output voltage across the resistive load \( V(t) = \dot{\lambda}(t) \)
Bistable oscillators vs resonant systems for vibration energy harvesting

Buckled piezoelectric beams

FEA time-dependent simulations with sinusoidal excitation

contracting the clamping of 0.1mm we induce snap-through bistable response

Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental apparatus (NiPS Lab. Perugia)

1) Laser displacement sensors
2) Shaker
3) Spectrum analyzer
4) Acquisition systems, signal generators and power amplifier
5) Piezoelectric beam
6) Accelerometer
7) Micrometric stage
Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental test

![Noise Power Spectral Density](chart)


\[
\langle \xi_e(t) \xi_e(t') \rangle = \langle \xi_e^2 \rangle \exp \left[ -|t-t'|/\tau_c \right] \quad \tau = 0.1 - 0.001s
\]

In the postbuckled static configuration the elastic energy results to be

\[
\Pi(q) = \frac{1}{4} k_3 q^4 + \frac{1}{2} k_2 q^2 + P_{cr} d_b.
\]
Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental and numerical results

Snapping between buckled states

(a) Load Resistance = 1MΩm

(b) Load Resistance = 25kΩm

(c) Voltage (V)

(d) Voltage (V)
Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental and numerical results

Snapping between buckled states

Power spectral density
Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental and numerical results

Submitted for publication to Smart Materials & Structures journal:
“Piezoelectric buckled beams for random vibrations energy harvesting”
F Cottone, L Gammaitoni, H Vocca, M Ferrari, V Ferrari
Bistable oscillators vs resonant systems for vibration energy harvesting

Experimental and numerical results

Submitted for publication to Smart Materials & Structures journal:
“Piezoelectric buckled beams for random vibrations energy harvesting”
F Cottone, L Gammaitoni, H Vocca, M Ferrari, V Ferrari
Buckled beam concept for other transduction methods

Electromagnetic

Electrostatic/Capacitive

Magnetostriective

Piezoelectric

Ferroelectric materials: PZT, PVDF, AIN

Ferromagnetic materials: crystalline alloy Terfenol-D amorphous metallic glass Metglas (Fe$_8$B$_{13.5}$Si$_{3.5}$C$_2$).
Buckled beam concept for other transduction methods

Silicon-based MEMS comb-drive capacitor

Electromagnetic buckled beam system

NEHSTech IEF Marie Curie Project at ESIEE
University of Paris Est - prof. P. Basset and F. Cottone
Conclusions

• Bistable harvesters have been confirmed to outperform resonant systems under random and harmonic vibration source both in terms of frequency response and voltage amplitude.

• A buckled piezoelectric beam has been theoretically modeled, also considering in-plane stretching effects, and experimentally investigated.
  • Numerical results predict in good qualitative agreement the physical behaviour.

• The nonlinear buckled configuration has been demonstrated to be more efficient in terms of power density and frequency bandwidth. The overall electrical power results multiplied by more than a factor 3x.

• A counterintuitive not decreasing voltage occurs even when the systems oscillates within one of the two minima at high compression load.
Future work

• Validation of the buckled beam concept is envisaged for other type of conversion methods: electrostatic, electrodynamic and/or magnetostrictive.

• Scaling down to millimetric- and/or micro-scale must be further investigated.

• Vibration-driven microgenerator must be tested in real scenarios to assess the efficacy of the proposed concept. For example by using real vibration database.

• Integrated wireless sensor node system with power aware electronics and nonlinear vibrational generators are expected to be developed ad validated.
Thanks for your attention!